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# The disturbance caused in heat due to presence of two Griffith-Cracks opened by two exterior wedges.

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**ABSTRACT**
The closed form expression of disturbance in temperature distribution in a rectangular isotropic in the presence of two Griffith-cracks opened by two exterior wedges is being obtained by the principal of cross linear superposition along with Fredholm integral equation. It is found that temperature distribution at crack tips is smooth. Flux possesses Cauchy type of singularity at crack tips.
**Key-Word:-** [ Flux intensity factor, Fredholm integral , Cross linear superposition]

## I. INTRODUCTION

We consider the problem of single and double Griffith-cracks and got dual and triple series equations which were solved by the method of Parihar [1]. It is very obvious that will happen if there are two Griffith-cracks opened by two exterior wedges. As we know this problem will reduce to Triple series equation. We consider a cross section of three dimensional body having two Griffith-Crack along x-axis and y-axis being through two exterior wedges cracks and perpendicular to x-axis. Thus we consider a rectangle of length 2a and width 2δ. The physically problem will be reduced to the following boundary value problem for steady case.

$$\frac{\partial}{\partial y}T(x,\delta)=Q_1(x) \quad , \ 0\le x\le a \qquad 1.1$$

$$\frac{\partial}{\partial x}T(a,y)=Q_2(x) \quad , \ 0\le y\le \delta \qquad 1.2$$

$$\frac{\partial}{\partial y}T(x,0)=Q_3(x) \quad , \ b<x<c \qquad 1.3$$

$$T(x,0)=\begin{cases} T_0(x) & ,c\le x\le a \\ T_1(x) & ,0\le x\le b \end{cases} \qquad 1.4$$

Where the symmetry of geometry is used. The temperature T satisfy

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)T(x,y)=0 \qquad 1.5$$

The plan of paper as follows. We will formulate and reduce the problem to triple series equation. The solution of triple series equation will be obtained. Further we will solve the physical quantities in term of Fredholm integral equation. The solution of Fredholm integral equation will be given in section.

**Formulation**

We take the solution of (1.5) and then satisfy the boundary conditions in (1.1) and (1.2) gives

$$\alpha_n[A_n \sinh(\alpha_n\delta) + B_n \cosh(\alpha_n\delta)] = F_1(\alpha_n)$$

2.1

$$\beta_m c_m \sinh(\alpha\beta_m) = F_2(\beta_m)$$

2.2

Where

$$F_1(\alpha_n) = \frac{2}{a}\int_0^a Q_1(x)\cos(\alpha_n x)dx$$

$$F_2(\beta_m) = \frac{2}{\delta}\int_0^\delta Q_2(y)\cos(\beta_m y)dy$$

2.3

The mixed boundary condition gives :-

$$\frac{A_0 + C_0}{2} + \sum_{n=1}^{\infty} A_n \cos(\alpha_n x) = \begin{cases} P_0(x) & ,0 \le x \le b \\ -P_1(x) & ,c \le x \le a \end{cases}$$

2.4

$$\sum_{n=1}^{\infty} \alpha_n A_n \cos(\alpha_n x) = P_2(x) - 2\sum_{n=1}^{\infty} e^{-2\alpha_n\delta}\alpha_n \cos(\alpha_n x) \quad ,b < x < c$$

2.5

$$P_3(x) = -P_1(x) + T_1(x)$$

2.6

$$P_1(x) = \sum_{m=1}^{\infty} \frac{F_2(\beta_m)\cosh(\beta_m x)}{(\beta_m \sinh \beta_m \alpha)} + T_0(x)$$

2.7

$$P_2(x) = -Q_3(x) + \sum_{n=1}^{\infty} \frac{F_1(\alpha_n)\cosh(\alpha_n x)}{\cosh(\alpha_n\delta)}$$

2.8

The solution of triple series (2.4)-(2.5) will be given below

**Solution of Triple Series**

We assume trial solution as

$$\alpha_n A_n = 2\left[\left(\int_b^c g(t) - \frac{1}{a}\int_0^b P_3^{'}(t) + \frac{1}{a}\int_c^a P_1^{'}(t)\right)\sin(\alpha_n t)dt\right]$$

2.9

$$\frac{A_0 + C_0}{2} = \left[\left(\int_b^c tg(t)dt - \frac{1}{a}\int_0^b tP_3^{'}(t)dt + \frac{1}{a}\int_c^a tP_1^{'}(t)dt\right) - P_1(a)\right]$$

2.10

Then substitution of (2.9)-(2.10) into (2.4) satisfies it identically if

$$\int_b^c g(t)dt = \frac{P_3(b)+P_1(c)}{a}$$
2.11

Thus substituting (2.9) into (2.5) and inverting

$$g(t) = \frac{1}{a^2\delta(t)}\left[\Delta_0(t) + \int_b^c g(\alpha).k(\alpha,t)d\alpha\right] \qquad ,b \le t < c$$
2.12

$$\Delta_0(t) = \int_b^c \frac{\sin(qx)\delta(x)P_4(x)}{G(x,t)}dx + D$$
2.13

Where D is arbitrary constant to be determine through (2.11)

$$P_4(x) = P_2(x) + \frac{1}{a}\left(\int_0^b P_3^{'}(\alpha) - \int_c^a P_1^{'}(\alpha)\right)P_{41}(\alpha,x)dt$$
2.14

$$P_{41}(\alpha,x) = \frac{\sin(qx)}{G(\alpha,x)} + M(\alpha,x)$$
2.15

$$k(\alpha,t) = \frac{\sin(qx)\delta(x)}{G(x,t)}M(\alpha,x)dx$$
2.16

$$M(\alpha,x) = \sum_{n=1}^{\infty} e^{-2\alpha_n\delta}(1+e^{-2\alpha_n\delta})^{-1}\sin(\alpha\alpha_n)\cos(\alpha_n x)$$
2.17

$$f(t) = \left\{\left|G(t,b)G(t,c)\right|\right\}^{\frac{1}{2}} \text{ , } G(t,b) = \cos qt - \cos qb$$
2.18

The (2.13) is called Fredholm integral equation of second kind.

**Physical Quantities – Temperature**

The temperature distribution over crack faces is the value of $T(x,0)$ $for\ b < x < c$

$$T(x,0) = a\int_x^c g(t)dt - P_1(c)$$
3.1

Considering

$$\lim_{x \to b^+} T(x,0) = a\int_b^c g(t)dt - P_1(c) = P_3(b)$$
3.2

And

$$\lim_{x \to c^-} T(x,0) = -P_1(c)$$
3.3

The limits in (3.2)-(3.3) give the values of temperature as it should be . Thus we see that temperature distribution is smooth at crack tips.

**Flux**

Flux cross x-axis is $\frac{\partial}{\partial y}T(x,0)$ *and is given as*

$$\frac{\partial}{\partial y}T(x,0)=\sum_{n=1}^{\infty}\alpha_n A_n\cos(\alpha_n x)+2\sum_{n=1}^{\infty}\alpha_n e^{-2\alpha_n\delta}(1+e^{-2\alpha_n\delta})^{-1}$$

$$A_n\cos(\alpha_n x)=P_5(x)$$

$$P_5(x)=\sum_{n=1}^{\infty}\frac{F_1(\alpha_n)\cos(\alpha_n x)}{\cosh(\alpha_n\delta)}$$

3.4

Now use (2.9) in (3.4) we get

$$\frac{\partial}{\partial y}T(x,0)=\int_b^c\frac{g(t)\sin(qt)}{G(x,t)}dt+\int_b^c g(\alpha)M(\alpha,x)d\alpha-P_5(x)$$

3.5

$$M(\alpha,x)=\sum_{n=1}^{\infty}\frac{e^{-\alpha_n\delta}\cos(\alpha_n x)\sin(\alpha\alpha_n)}{\cosh(\alpha_n\delta)}$$

3.6

Now we evaluate the integral in first term of right hand side of (3.5) after substituting the value of g(t) from (2.12)

$$\frac{\partial}{\partial y}T(x,0)=\begin{cases}\dfrac{\Delta_2(x)}{\alpha\delta(x)}+P_6(x) & ,\ 0\le x\le b\\ \dfrac{-\Delta_1(x)}{\alpha\delta(x)}+P_6(x) & ,c<x\le a\end{cases}$$

3.7

$$P_6(x)=\int_b^c g(\alpha)M(\alpha,x)d\alpha-P_5(x)$$

3.8

$$\Delta_2(x)=\Delta_0(x)+\int_b^c g(\alpha)k(\alpha,x)d\alpha$$

3.9

Thus we see the flux possesses the Cauchy type of singularity at crack tips.

**Flux Intensity Factor**

The flux intensity factor at crack tips is defined as

$$F_b=\lim_{x\to b^-}\sqrt{x-b}\,\frac{\partial}{\partial y}T(x,0)$$

$$F_c=\lim_{x\to c^+}\sqrt{c-x}\,\frac{\partial}{\partial y}T(x,0)$$

3.10

Now evaluating the limits in (3.10) after using (3.7)-

$$F_b=\sqrt{\psi_0\frac{b}{\pi a}.\Delta_2(b)}$$

3.11

$$F_b = -\sqrt{\psi_0 \frac{c}{\pi a}}.\Delta_2(c)$$

3.12

Where $\Delta_2(x)$ , $x = b, c$ is given by (3.9). $P_6(x)$ does not possesses singularity in crack tips.

In next section we shall consider the special type boundary conditions and solve the Fredholm integral equations

**Solution of Fredholm Integral Equation**

Before we solve Fredholm integral equation , we assume the boundary conditions as

$$Q_1(x) = R_0 = Cons\tan t \quad , \; Q_2(y) = 0$$

4.1

The edges parallel to crack axis at constant flux while edges perpendicular to crack axis are insulated

$$Q_3(x) = R_1 = Cons\tan t \; , T_0(x) = R_2 = Cons\tan t \, , T_1(x) = R_3 = Cons\tan t$$

4.2

The constants are such that $R_1 > R_2 > R_3 > R_0$ 4.3

Use (4.3) in (2.3) we get

$$F_1(\alpha_n) = F_2(\beta_m) = 0$$ 4.4

Thus $P_1(x) = R_2$ , $P_2(x) = -R_1$ , $P_3(x) = R_3 - R_2$ 4.5

The condition (2.11) becomes

$$\int_b^c g(t)dt = \frac{R_3}{a}$$

4.6

$$P_4(x) = -R_1 = Cons\tan t$$

4.7

$$\Delta_0(x) = -a\,R_1(G(b,t)) \quad , \; b < t < c$$

4.8

$$P_5(x) = 0$$

4.9

We approximate $M(\alpha, x)$ *and* $K(\alpha, t)$ as given in (2.16) and (2.15), according to the method used by Kushwaha last paper [2].

$$M(\alpha, x) = \sum_{k=2,4,6}^{\infty} (-1)^{\frac{k}{2}} \psi_k(\alpha, x)$$

4.10

$$\psi_k(\alpha, x) = \frac{2\sin(q\alpha)\cos(qx)}{\cosh(kq\delta)} + \frac{\sin(2q\alpha)\cos(2q\alpha)}{\cosh^2(kq\delta)} + \frac{1}{\cosh^2(kq\delta)}\left[\frac{3}{2}\sin(q\alpha)\cos(qx) + \frac{1}{3}\sin(3q\alpha)\cos(3qx)\right]$$

4.11

$$k(\alpha, t) = \sum_{R=2,4,6}^{\infty} \frac{\sin(q\alpha)}{\cosh(kq\delta)} I_1(t) + \frac{\sin(2q\alpha)}{\cosh^2(kq\delta)} M_1(t) + \frac{1}{\cosh^3(kq\delta)}\left[\frac{3}{2}\sin(q\alpha) I_1(t) + \frac{1}{3}\sin(3q\alpha) M_2(t)\right](-1)^{\frac{k}{2}}$$

4.12

$$M_1(t) = 2I_2(t) - I_0(t) \; , I_0(t) = aG(b,t) \; , \; M_2(t) = 4I_3(t) - 3I_1(t)$$

4.12a

$$I_n(t)=\int_b^c \frac{\sin(qx)\delta(x)\cos(qx)}{G(x,t)}dx \qquad ,b<t<c$$

$$I_n(t)=T_{n-1}+\cos(qt)I_{n-1}(t)$$

4.12b

$$T_n=4an!\sum_{n=0}^{\infty}\frac{(qc)^{n-r}(G(b,c))^{r+1}}{(n-r)!(r+2)!(2r+1)!}$$

4.12c

We assume g(t) as

$$g(t)=\sum_{r=0}^{\infty} g_r(t)\cosh(2q\delta)^{-r} \qquad ,b<t<c \tag{4.13}$$

Then using (4.13)-(4.12) in (2.12) and comparing the coefficients $(\cos 2qr)^{-r}$ , $r=0,1,2,3$ only from both sides we get

$$g_0(t)=\frac{\Delta_0(t)}{a^2\delta(t)} \quad , b<t<c \tag{4.14}$$

$$\Delta_0(t)=aR_1G(b,t)+D$$

4.14a

$$D=\frac{R_3-R_1E\left(\frac{\pi}{2},\mu_1\right)\cos ec\left(\frac{qc}{2}\right)}{\sin\left(\frac{qb}{2}\right)F\left(\frac{\pi}{2},\mu_0\right)}$$

4.14b

$$\mu_0{}^2=\frac{G(b,c)}{\sin^2\left(\frac{qb}{2}\right)} \quad , \quad \mu_1{}^2=\frac{G(b,c)}{\sin^2\left(\frac{qc}{2}\right)}$$

4.14c

E, F are complete elliptic integrals of First and Second type.

$$g_1(t)=\frac{-2I_1(t)}{a^4\delta(t)} \qquad , \ a_0=R_1G(b,c)+\frac{D}{a} \tag{4.15}$$

$$g_2(t)=\frac{2a_0}{a^4\delta(t)}\left[a_2M_1(t)-\frac{2a_1I_1(t)}{a^2}.a_3\right] \tag{4.16}$$

$$g_3(t)=\frac{a_0}{a^4\delta(t)}\left[\frac{3a_1}{2}I_1(t)+\frac{a_n}{2}M_2(t)+2a_5M_1(t)+2a_6I_1(t)\right] \tag{4.17}$$

Thus $g(t)=\dfrac{\Delta_2(t)}{a^2\delta(t)} \qquad , b<t<c$ 4.18

$$\Delta_2(t)=\left[\begin{array}{l} R_1aG(b,t)+D-\dfrac{a_0}{\cosh 2q\delta}\left(\dfrac{I_1(t)}{a^2}-\dfrac{2}{\cosh 2q\delta}\left\langle a_2M_1(t)-\dfrac{2a_1a_3}{a^2}I_1(t)\right\rangle\right) \\ -\dfrac{1}{\cosh^2 2q\delta}\left\langle\dfrac{3a_1}{2}I_1(t)+\dfrac{a_4}{2}M_2(t)+2a_5M_1(t)+2a_2I_1(t)\right\rangle \end{array}\right] \text{, b<t<c} \quad 4.18a$$

It is observed that g(t) depends upon the temperature over we wedge and fluc crack faces and not on edges parallel to crack axis or the temperature over crack axis with 0<x<b. The temperature distribution over crack faces is given by equation (3.1) and (4.18)Integral can easily be written or can easily be evaluated numerically.The flux intensity factor at crack tips are given by (3.11) and $\Delta_2(t)$ from (4.18a).

## II. DISCUSSION AND CONCLUSION

It is observed that g(t) depends upon the temperature over the wedge and flux upon crack faces and not on edges parallel to crack axis or the temperature over crack axis with $0 \leq x \leq b$. It is observed that the temperature over crack is smooth while flux or flux intensity factor will for plastic zone in the neighborhood of crack tips. It has also Cauchy type of singularity.

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