International Journal of Advances in Engineering and Management (IJAEM)
Volume 3, Issue 10 Oct 2021, pp: 126-132 www.ijaem.net ISSN: 2395-5252

"The New Integral Transform "Soham Transform"

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Submitted: 25-09-2021 Revised: 01-10-2021 Accepted: 05-10-2021

ABSTRACT: In this paper a new integral transform namely Soham transform is developed and applied to solve linear ordinary differential equations with constant coefficients.

Keywords: SohamTransform,Differential Equations, Integral Transform.

I. INTRODUCTION :

S.S Khakale and D.P Patil are gotten from the old-style Laplace indispensable. In light of the numerical effortlessness of the Soham and its principal properties.

Soham transform is introduced by S.S. Khakale and D.P. Patil to facilitate the process of solving ordinary and partial differential equations in the time domain. Regularly, Fourier , Elzaki,Aboodh, Kamal, Mohand transforms are the convenient mathematical tools for solving differential equations, Also Soham transform and some of its fundamental properties are used to solve differential equations.

Many researchers has developed lot of integral transforms like Aboodh[7],Sumudu[2].Author [5] compared Laplace, Sumudu and Mahagoub transform in 2018. D.P. Patil [5] compared Laplace, Sumudu, Aboodh and Elazaki and Mahagoub transform in 2018. Further D.P. Patil used Aboodh and Mahagoub transform in boundary value problems of system of ordinary differential equation. He [8] apply sawi transform in Bessels function Sawi transform is applied for wave equation [9], [10] and error function for evaluating improper integrals.

A new transform called the Soham transformdefined for function of exponential order we consider functions in the set B defined by:

$B = \{f(t): \exists M, k_1, k_2 > 0. |f(t)| < Me^{|t|k_j}, \text{if } t \in (-1)^j \times \{0,\infty)\}$ (1)

For a given function in the set B, the constant M must be finite number, , k_1, k_2 may be finite or infinite.

Soham transform denoted by the operator $\mathcal{S}(.)$ defined by the integral equations

$\mathcal{S}[f(t)] = P(v) = \frac{1}{v}\int_0^\infty f(t)e^{-v^\alpha t}dt$, α is non zero real numbers $t \geq 0,\ k_1 \leq v \leq k_2$ (2)

The variable v in this transform is used to factor the variable t in the argument of the function f this transform has deeper connection with Aboodh, Sumudu, and the new integral Transform. The purpose of this study is to show the applicability of this interesting new transform and its efficiency in solving the linear differential equations.

Notes:

(1) If $\alpha = 1$then equation(2) becomes:

$$\mathcal{S}[f(t)] = P(v) = \frac{1}{v}\int_0^\infty f(t)e^{-vt}dt \quad t \geq 0,\ k_1 \leq v \leq k_2$$ This integral transform is called "Aboodh Transform"

(2) If $\alpha = -1$then equation(2) becomes:

$$\mathcal{S}[f(t)] = P(v) = \frac{1}{v}\int_0^\infty f(t)e^{-\frac{1}{v}t}dt \quad t \geq 0,\ k_1 \leq v \leq k_2$$ This integral transform is called "Sumudu Transform"

(3) If $\alpha = -2$then equation(2) becomes:
$\mathcal{S}[f(t)] = P(v) =$
$\frac{1}{v}\int_0^\infty f(t)e^{-\frac{1}{v^2}t}dt \quad t \geq 0, \; k_1 \leq v \leq k_2$ This integral transform is called "The new integral Transform" Obtain in 2013.

1. Inverse Soham Transform:

Inverse Soham transform is denoted as follows:
Inverse Soham transform of f(t) is P(v) then inverse Soham transform is defined as

$$\mathcal{S}^{-1}[P(v)] = f(t)$$

2. SOHAM TRANSFORM OF THE ELEMENTARY FUNCTIONS:

For any function f(t), we assume that the integral equation (2) exist. The Sufficient Conditions for the existence of Soham transform are that f(t) for $t \geq 0$ be piecewise continuous and of exponential order, Otherwise Soham transform may or may not exist.
In this section we find Soham transform of simple functions.

(i) Let f(t)=1, then , By using definition we get,

$$\mathcal{S}[1] = P(v) = \frac{1}{v}\int_0^\infty 1.e^{-v^\alpha t}dt = \frac{1}{v}\int_0^\infty e^{-vt}\,dt = \frac{1}{v}[-\frac{1}{v^\alpha}e^{-vt}]_0^\infty = \frac{1}{v^{\alpha+1}} \qquad t \geq 0,$$

(ii) Let f(t)=t then ,
(iii) $\mathcal{S}[t] = P(v) = \frac{1}{v}\int_0^\infty t.e^{-v^\alpha t}\,dt$
Integrating by parts, we get $\mathcal{S}[t] = \frac{1}{v^{2\alpha+1}}$
In the general case if $n = 0,1,2,3, \dots \dots \dots \dots$ is integer number, then,

$$\mathcal{S}[t^n] = \frac{\Gamma(n+1)}{v^{\alpha n+\alpha+1}}$$

(iv) $\mathcal{S}[e^{at}] = \frac{1}{v}\int_0^\infty e^{at}e^{-v^\alpha t}dt = \frac{1}{v(v^\alpha - a)}$ and $\mathcal{S}[e^{-at}] = \frac{1}{v}\int_0^\infty e^{-at}e^{-vt}dt = \frac{1}{v(v^\alpha + a)}$
This result will be useful, to find Soham transform of:
$\mathcal{S}[sinat] = \frac{a}{v(v^{2\alpha}+a^2)}$, $\mathcal{S}[cosat] = \frac{v^\alpha}{v(v^{2\alpha}+a^2}$
$\mathcal{S}[sinhat] = \frac{av}{v^{2\alpha}-a^2}$, $\mathcal{S}[coshat] = \left(\frac{v^\alpha}{v^{2\alpha}-a^2}\right)$

Linearity Property of Soham Transform:
Theorem2.1: If $f_1(t) and\, f_2(t)$ be two functions of t and c_1 and c_2 be any two constants then $\mathcal{S}\{c_1 f_1(t) + c_2 f_2(t)\} = c_1\mathcal{S}\{f_1(t)\} + c_2\mathcal{S}\{f_2(t)\}$.
Proof:
Let$f_1(t) and\, f_2(t)$ be two functions of t and c_1 and c_2 be any two constants. We have by using definition of Soham transform,

$$\mathcal{S}\{c_1 f_1(t) + c_2 f_2(t) = \frac{1}{v}\int_0^\infty e^{-v^\alpha t}[c_1 f_1(t) + c_2 f_2(t)]dt$$
$$= \frac{1}{v}\int_0^\infty e^{-v^\alpha t}c_1 f_1(t) + \frac{1}{v}\int_0^\infty e^{-v^\alpha t}\,c_2 f_2(t)]dt$$
$$= c_1\frac{1}{v}\int_0^\infty e^{-v^\alpha t}f_1(t) + c_2\frac{1}{v}\int_0^\infty e^{-v^\alpha t}\,f_2(t)]dt$$
$$= c_1\mathcal{S}\{f_1(t)\} + c_2\mathcal{S}\{f_2(t)\}$$

The proof is complete

Theorem2.2:
Let $P(v)$ Soham transform of $[\mathcal{S}[f(t)] = P(v)]$ then:
(i) $\mathcal{S}[f'(t)] = v^\alpha P(v) - \frac{1}{v}f(0)$
(ii) $\mathcal{S}[f''(t)] = v^{2\alpha}P(v) - v^{\alpha-1}f(0) - \frac{1}{v}f'(0)$

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(iii) $\mathcal{S}[f^n(t)] = v^{n\alpha} P(v) - \frac{1}{v}\sum_{k=0}^{n-1} v^{\alpha(n-1-k)} f^k(0)$

Proof:

By using definition of soham transform, we get,

$\mathcal{S}[f'(t)] = \frac{1}{v}\int_0^\infty f'(t)e^{-v^\alpha t}\,dt,$

Integration by parts gives,

$$\mathcal{S}[f'(t)] = \frac{1}{v}\left\{\left[e^{-v^\alpha t} f(t)\right]_0^\infty - \int_0^\infty -v^\alpha\, e^{-v^\alpha t} f(t)dt\right\}$$

$$= \frac{1}{v}\left\{[0 - f(0)] + v^\alpha \int_0^\infty e^{-v^\alpha t}\, f(t)dt\right\}$$

$$= -\frac{1}{v} f(0) + v^\alpha \frac{1}{v}\int_0^\infty e^{-v^\alpha t}\, f(t)dt$$

$$= v^\alpha P(v) - \frac{1}{v} f(0)$$

Therefore

$$\mathcal{S}[f'(t)] = v^\alpha P(v) - \frac{1}{v} f(0)$$

Let $g(t) = f'(t)$, then

$$\mathcal{S}[g'(t)] = v^\alpha\{\mathcal{S}[f'(t)]\} - \frac{1}{v} f'(0)$$

We find that by using (i), we get,

$$\mathcal{S}[f''(t)] = v^{2\alpha} P(v) - v^{\alpha-1} f(0) - \frac{1}{v} f'(0)$$

(i) Can be Proof by Mathematical induction then

$$\mathcal{S}[f^n(t)] = v^{n\alpha} P(v) - \frac{1}{v}\sum_{k=0}^{n-1} v^{\alpha(n-1-k)} f^k(0)$$

3. Application Of Soham Transform Ofordinary Differential Equations

As stated in the introduction of this paper, the Soham transform can be used as an effective tool. For analyzing the basic characteristics of a linear system governed by the differential equation in response to initial data. The following examples illustrate the use of the Soham transform in solving certain initial value problems described by ordinary differential equations.

Case-I: Linear differential equation of first order general form initial value problem:

$\frac{dx}{dt} + px = f(t), \quad t > 0$ (3)

With the initial condition

$x(0) = a$ (4)

Where p and a are constants and $f(t)$is an external input function so that its Soham transform exists.

We Apply Soham transform to differential Equation (3), we obtain,

$$\mathcal{S}\left(\frac{dx}{dt}\right) + \mathcal{S}(px) = \mathcal{S}(f(t))$$

$$\mathcal{S}\left(\frac{dx}{dt}\right) + p\mathcal{S}(x) = \mathcal{S}(f(t)$$

$$v^\alpha P(v) - \frac{1}{v} x(0) + pP(v) = \bar{f}(v)$$

$$(v^\alpha + p)P(v) - \frac{1}{v} x(0) = \bar{f}(v)$$

Using initial condition of above Equation, we get,

$$P(v) = \frac{\bar{f}(v) + a\frac{1}{v}}{v^\alpha + p} = \frac{v\bar{f}(v) + a}{v(v^\alpha + p)}$$

We apply inverse Soham transform of above equation leads to the solution.

Case-II The Second order linear ordinary differential equation of initial value problem having the general form:

$\frac{d^2y}{dx^2} + 2p\frac{dy}{dx} + qy = f(x), \qquad x > 0 \quad (5)$

The initial conditions are

$y(0) = a, \qquad y'(0) = b(6)$

Where p, q, a and b are constants.We apply Soham transforms to differential equation (5), and we obtain,

$$\mathcal{S}\left(\frac{d^2y}{dx^2}\right) + 2p\mathcal{S}\left(\frac{dy}{dx}\right) + q\mathcal{S}(y) = \mathcal{S}(f(x))$$

$$\left(v^{2\alpha}P(v) - v^{\alpha-1}y(0) - \frac{1}{v}y'(0)\right) + 2p\left(v^{\alpha}P(v) - \frac{1}{v}y(0)\right) + qP(v) = \bar{f}(v)$$

After Simplification, we get,

$$P(v) = \frac{\bar{f}(v) + \frac{1}{v}y'(0) + \left(v^{\alpha-1} + 2p\frac{1}{v}\right)y(0)}{(v^{2\alpha} + 2pv^{\alpha} + q)}$$

$$P(v) = \frac{\bar{f}(v)}{(v^{2\alpha} + 2pv^{\alpha} + q)} + \frac{y'(0)}{v(v^{2\alpha} + 2pv^{\alpha} + q)} + \frac{\left(v^{\alpha-1} + 2p\frac{1}{v}\right)y(0)}{(v^{2\alpha} + 2pv^{\alpha} + q)}$$

The use the initial condition of Equation (6) leads to the solution for $P(v)$ as

$$P(v) = \frac{\bar{f}(v)}{(v^{2\alpha} + 2pv^{\alpha} + q)} + \frac{b}{v(v^{2\alpha} + 2pv^{\alpha} + q)} + \frac{\left(v^{\alpha-1} + 2p\frac{1}{v}\right)a}{(v^{2\alpha} + 2pv^{\alpha} + q)}$$

We apply the inverse Soham transform of above equation gives the solution.

Example 3.1:

Consider the first order differential equation

$\frac{dy}{dt} + y = 0, \qquad y(0) = 1 \quad (7)$

We apply the Soham transform to both sides of this equation and using the differential property of Soham transform, Equation (7) can be written as:

$$v^{\alpha}P(v) - \frac{1}{v}y(0) + P(v) = 0$$

Using initial condition of above equation, we get,

$$(v^{\alpha} + 1)P(v) = \frac{1}{v}$$

$$P(v) = \frac{1}{v(v^{\alpha} + 1)}$$

Now applying inverse Soham transform of this equation gives the solution:

$$y(x) = e^{-x}$$

Example 3.2:

Solve the differential equation

$\frac{dy}{dt} + 2y = x, \qquad y(0) = 1 \quad (8)$

Applying Soham transform to both sides of above equation (8) and using the differential property of Soham transform, Equation (8) can be written as:

$$v^{\alpha}P(v) - \frac{1}{v}y(0) + 2P(v) = \frac{1}{v^{2\alpha+1}}$$

$$(v^{\alpha} + 2)P(v) = \frac{1}{v^{2\alpha+1}} + \frac{1}{v}y(0)$$

Using initial conditions and $P(v)$ is Soham transform, we get

$$P(v) = \frac{1}{v^{2\alpha+1}(v^{\alpha} + 2)} + \frac{1}{v(v^{\alpha} + 2)}$$

$$P(v) = \frac{1}{2}\frac{1}{v^{2\alpha+1}} + \frac{5}{4}\frac{1}{v(v^{\alpha} + 2)} - \frac{1}{4}\frac{1}{v^{n+1}}$$

The inverse Soham transform of this equation gives the solution:

$$y(x) = \frac{1}{2}x + \frac{5}{4}e^{-2x} - \frac{1}{4}$$

Example 3.3:

Let us consider the second-order differential equation

$y'' + y = 0\,, \qquad y(0) = y'(0) = 1$ (9)

Applying Soham transform to both sides of equation(9) and using the differential property of Soham transform, Equation(9) can be written as:

$$v^{2\alpha}P(v) - v^{\alpha-1}y(0) - \frac{1}{v}y'(0) + P(v) = 0$$

$$(v^{2\alpha} + 1)P(v) = v^{\alpha-1}y(0) + \frac{1}{v}y'(0)$$

Using initial condition of equation, we get,

$$(v^{2\alpha} + 1)P(v) = v^{\alpha-1} + \frac{1}{v}$$

$$P(v) = \frac{v^{\alpha} + 1}{v(v^{2\alpha} + 1)} = \frac{v^{\alpha}}{v(v^{2\alpha} + 1)} + \frac{1}{v(v^{2\alpha} + 1)}$$

The inverse Soham transform of this equation gives the solution:

$$y(x) = cos(x) + sin(x)$$

Therefore

$$y(x) = sin(x) + cos(x)$$

Example 3.4:

Consider the following equation

$y'' - 3y' + 2y = 0, \qquad y(0) = 1,\ y'(0) = 4$ (10)

Applying Soham transform of Equation (10) and using the differential property of Soham transform, Equation (10) we find that:

$$v^{2\alpha}P(v) - v^{\alpha-1}y(0) - \frac{1}{v}y'(0) - 3\left[v^{\alpha}P(v) - \frac{1}{v}y(0)\right] + 2P(v) = 0$$

$$(v^{2\alpha} - 3v^{\alpha} + 2)P(v) = v^{\alpha-1}y(0) + \frac{1}{v}y'(0) - 3\frac{1}{v}y(0)$$

Using initial condition of above equation, we get,

$$P(v) = \frac{v^{\alpha} + 4 - 3}{v(v^{2\alpha} - 3v^{\alpha} + 2)} = \frac{v^{\alpha} + 1}{(v^{\alpha} - 2)(v^{\alpha} - 1)}$$

$$= \frac{3}{v(v^{\alpha} - 2)} - \frac{2}{v(v^{\alpha} - 1)}$$

Then the solution is

$$y(x) = 3e^{2x} - 2e^{x}$$

Example 3.5:

Let the Second order differential equation:

$y'' + 9y = cos2t, \qquad y(0) = 1,\ \ y\left(\frac{\pi}{2}\right) = -1$ (11)

Since $y'(0)$ is not known, let $y'(0) = c$.

Applying Soham transform of this equation (11) and using the differential property of Soham transform, Equation (11) we find that:

$$v^{2\alpha}P(v) - v^{\alpha-1}y(0) - \frac{1}{v}y'(0) + 9P(v) = \frac{v^{\alpha}}{v(v2^{\alpha} + 4)}$$

$$(v^{2\alpha} + 9)P(v) = \frac{v^{\alpha}}{v(v^{2\alpha} + 4)} + v^{\alpha-1}y(0) + \frac{1}{v}y'(0)$$

Using initial condition of above equation, we get,

$$(v^{2\alpha} + 9)P(v) = \frac{v^{\alpha}}{v(v^{2\alpha} + 4)} + v^{\alpha-1} + \frac{c}{v}$$

$$P(v) = \frac{v^{\alpha}}{v(v^{2\alpha} + 4)(v^{2\alpha} + 9)} + \frac{c}{v(v^{2\alpha} + 9)} + \frac{v^{\alpha-1}}{(v^{2\alpha} + 9)}$$

$$P(v) = \frac{4v^{\alpha}}{5v(v^{2\alpha} + 9)} + \frac{cv^{3}}{(v^{2\alpha} + 9)} + \frac{v^{\alpha}}{5v(v^{2\alpha} + 4)}$$

The inverse Soham transform of this equation gives the solution:

$$y(t)=\frac{4}{5}cos3t+\frac{c}{3}sin3t+\frac{1}{5}cos2t$$

To determine c note that $y\left(\frac{\pi}{2}\right)=-1$then we find c=$\frac{12}{5}$ then,

$$y(t)=\frac{4}{5}cos3t+\frac{4}{3}sin3t+\frac{1}{5}cos2t$$

Example 3.6:

Solve the differential equation:

$y''-3y'+2y=4e^{3t},\qquad y(0)=-3,\quad y'(0)=5$ (12)

Applying Soham transform of this equation (12) and using the differential property of Soham transform, Equation (12) we find that:

$$v^{2\alpha}P(v)-v^{\alpha-1}y(0)-\frac{1}{v}y'(0)-3\left[v^{\alpha}P(v)-\frac{1}{v}y(0)\right]+2P(v)=4\left(\frac{1}{v(v^{\alpha}-3)}\right)$$

$$(v^{2\alpha}-3v^{\alpha}+2)P(v)=4\left(\frac{1}{v(v^{\alpha}-3)}\right)+v^{\alpha-1}y(0)+\frac{1}{v}y'(0)-3\frac{1}{v}y(0)$$

Using initial condition of above equation, we get,

$$(v^2-3v+2)P(v)=4\left(\frac{1}{v(v^{\alpha}-3)}\right)-3v^{\alpha-1}+5\frac{1}{v}+9\frac{1}{v}$$

$$\mathrm{P(v)}=\frac{4}{\mathrm{v}(\mathrm{v}^{\alpha}-3)(\mathrm{v}^{\alpha}-2)(\mathrm{v}^{\alpha}-1)}-\frac{3\mathrm{v}^{\alpha-1}}{(\mathrm{v}^{\alpha}-2)(\mathrm{v}^{\alpha}-1)}+\frac{5}{\mathrm{v}(\mathrm{v}^{\alpha}-2)(\mathrm{v}^{\alpha}-1)}+\frac{9}{\mathrm{v}(\mathrm{v}^{\alpha}-2)(\mathrm{v}^{\alpha}-1)}$$

$$\mathrm{P(v)}=\frac{4}{\mathrm{v}(\mathrm{v}^{\alpha}-2)}+\frac{2}{\mathrm{v}(\mathrm{v}^{\alpha}-3)}-\frac{9}{\mathrm{v}(\mathrm{v}^{\alpha}-1)}$$

The inverse Soham transform of this equation gives the solution:

$$\mathrm{y(t)}=4.\mathrm{e}^{2\mathrm{t}}+2\mathrm{e}^{3\mathrm{t}}-9\mathrm{e}^{\mathrm{t}}$$

Example 3.7:

Find the solution of the following initial value problem:

$y''+4y=12t,\qquad y(0)=0,\quad y'(0)=7$ (13)

Applying Soham transform of this equation (13) and using the differential property of Soham transform, Equation (13) we find that:

$$\mathrm{v}^{2\alpha}\mathrm{P(v)}-\frac{1}{\mathrm{v}}\mathrm{y}'(0)+4\mathrm{P(v)}=12\frac{1}{\mathrm{v}^{2\alpha+1}}$$

$$(\mathrm{v}^{2\alpha}+4)\mathrm{P(v)}=12\frac{1}{\mathrm{v}^{2\alpha+1}}+\frac{1}{\mathrm{v}}\mathrm{y}'(0)$$

Using initial condition of above equation, we get,

$$\mathrm{P(v)}=\frac{12\mathrm{v}+7\mathrm{v}^{2\alpha+1}}{\mathrm{v}^{2\alpha+2}(\mathrm{v}^{2\alpha}+4)}$$

$$\mathrm{P(v)}=3\frac{1}{\mathrm{v}^{2\alpha+1}}+\frac{4}{\mathrm{v}(\mathrm{v}^{2\alpha}+4)}$$

The inverse Soham transform of this equation gives the solution:

$$\mathrm{y(t)}=3\mathrm{t}+2\sin 2\mathrm{t}$$

CONCLUSION

The definition and application of the new transform "Soham transform" to the solution of ordinary differential equations has been demonstrated.

ACKNOWLEDGEMENTS

The authors grateful thanks to Principal Dr. D.B.Shinde for providing facilities for the research and Laboratory support in the preparation this revised paper.

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