

# Study 4WS System Data Map With The Effects Of Tire Stiffness

Ngoc Nguyen Van<sup>1</sup>, Long Huynh Diep Ngoc<sup>1</sup>, Duong Pham Thanh<sup>1</sup>, Tri Uong Hoang<sup>2</sup>  
*College of Technology II, Ho Chi Minh City, Vietnam<sup>1</sup>*  
*Thu Duc College Of Technology, Ho Chi Minh City, Vietnam<sup>2</sup>*

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## ABSTRACT :

The 4-wheel steering system or the all-wheel steering system (4WS - Four Wheel Steering) can control all wheels on the car, increasing the vehicle's stability at high speeds and reducing the steering angle at high speeds. low speed. The 4WS system was born in the 80s, equipped on Honda Preludes in the late 1980s and early 1990s.

The paper has built a kinematic relationship between the rotation angle of the 4WS guide wheels with the scaling factor  $k$ . Based on the built mathematical model, the author uses MATLAB/Simulink software to simulate the rotational dynamics of a car with a set of parameters of a specific vehicle. Perform the simulation of the rotation trajectory with different cases when changing the lateral stiffness of the tire.

The author has built the relationship between the angle of rotation of the rear guide wheel compared to the front according to the speed through an available data map.

**Keywords:** 4WS, Four Wheel Steering, Honda Prelude, ARS, 4WAS, four-wheel steering kinematics

## 1. Introduction

Since 1987, to improve stability when moving at high speed, there have been cars with structures that control the wheels to rotate in the same direction. Thanks to the combination with the solution to improve maneuverability when entering and leaving the parking space, and stability at high speeds, today's 4WS steering system has formed with 3 rear wheel control states as shown in [Fig. 1].

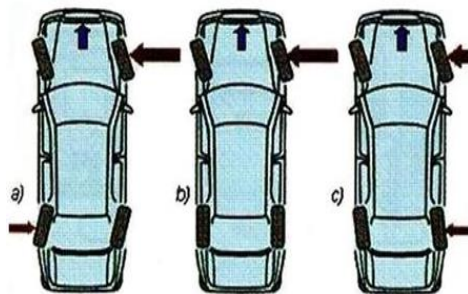


Fig 1. 4WS steering system with 3 rear wheel control modes

## Three basic control states in motion:

- When the steering wheel rotation angle is small and used at medium speed, the rear wheels lock up similar to the traditional structure [Fig 1. 1b].
- The front and rear wheels rotate in the opposite direction, this state is guaranteed to easily turn around, get in and out of the parking space, and have a small turning radius [Fig 1. 1a].
- The front and rear wheels rotate in the same direction, ensuring the vehicle's lack of rotation ability, that is, creating conditions to improve the stability of motion when operating at high speed [Fig 1. 1c].

From the characteristics of the above 4WS system arranged on current vehicles, to ensure the stability of the vehicle in unsafe conditions, rotational motion.

## 2. Model of changing the direction of two-track car movement 4WS

### 2.1. Assumptions when building models

Neglect the effect of vertical oscillation on the stability of the car's trajectory.

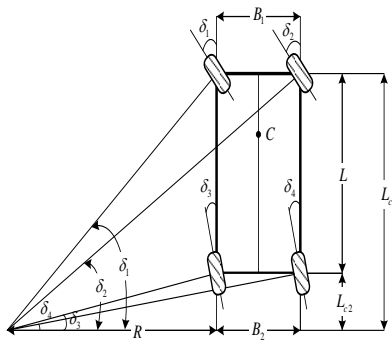
Neglect the effect of the vehicle body tilt (assume the road surface is flat).

The car's center of gravity is located on the road surface. Neglect the effect of tilting the body under the influence of centrifugal force and suspension. Cars move on four wheels.

Then the car's motion trajectory is a curve and is determined by consecutive positions of the car's center of gravity  $C$ .

### 2.2. Kinetic relationship of guide wheels

Consider the kinematic model of changing the direction of motion of the 4WS car shown in Fig 2. 5. The kinematic relationship of the rotation angle of the guide wheels. Inside:  $\delta_1, \delta_2, \delta_3, \delta_4$  is the rotation angle of the guide wheels;  $O$  is the center of rotation.



**Fig 2. Kinetic relation of the rotation angle of the guide wheels in the same direction [1]**

$$L_{c1} = L + L_{c2} \quad (1)$$

Inside:

L: the wheelbase of the vehicle;  $L_{c1}$ : the distance from the front axle to the alignment from the center of rotation perpendicular to the longitudinal axis of symmetry of the vehicle;  $L_{c2}$ : the distance from the rear axle to the alignment line from the center of rotation perpendicular to the longitudinal axis of symmetry of the vehicle. Put

$$L_{c2} = k \cdot L = k \cdot (L_{c1} - L_{c2}) \quad (2)$$

where k is the scaling factor.

$k > 0$ : increases the turning radius compared to when turning the two front wheels of the guide. Then the rear guide wheel rotates in the same direction as the front guide wheel.

$k < 0$ : reduces the turning radius compared to when turning the two front wheels of the guide. Then the rear guide wheel rotates in the opposite direction of the front guide wheel.

$k = 0$ : then there were only two front wheels guiding.

We have the distance from the front axle to the alignment from the center of rotation perpendicular to the longitudinal symmetry axis of the vehicle:

$$L_{c1} = L_{c2} + L = (1 + k) \cdot L = (1 + k) \cdot (L_{c1} + L_{c2}) \quad (3)$$

The correct turning condition is that the instantaneous centers of rotation of the guide wheels coincide at one point. From that condition, we have the correct rotation condition expression:

$$\cot g \delta_2 - \cot g \delta_1 = \frac{B_1}{L_{c1}} \quad (4)$$

$$\cot g \delta_4 - \cot g \delta_3 = \frac{B_2}{L_{c2}} \quad (5)$$

$$\frac{\text{tg} \delta_3}{\text{tg} \delta_1} = \frac{L_{c2}}{L_{c1}} \quad (6)$$

From (6) we have:

$$\frac{1}{\text{tg} \delta_2} - \frac{1}{\text{tg} \delta_1} = \frac{B_1}{L_{c1}} \\ \Rightarrow \frac{1}{\text{tg} \delta_2} = \frac{B_1}{L_{c1}} + \frac{1}{\text{tg} \delta_1} = \frac{B_1 \text{tg} \delta_1 + L_{c1}}{L_{c1} \text{tg} \delta_1}$$

$$\Rightarrow \delta_2 = \arctan \frac{L_{c1} \text{tg} \delta_1}{B_1 \text{tg} \delta_1 + L_{c1}} \quad \text{[rad]} \quad (7)$$

From (7) we have:

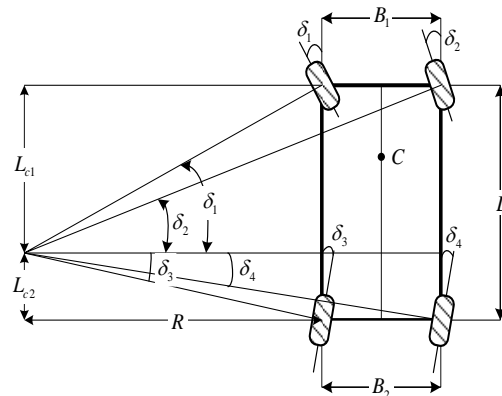
$$\Rightarrow \delta_3 = \arctan \left( \text{tg}(\delta_1) \cdot \frac{L_{c2}}{L_{c1}} \right) \quad \text{[rad]} \quad (8)$$

From (8) we have:

$$\frac{1}{\text{tg} \delta_4} - \frac{1}{\text{tg} \delta_3} = \frac{B_2}{L_{c2}} \\ \Rightarrow \frac{1}{\text{tg} \delta_4} = \frac{B_2}{L_{c2}} + \frac{1}{\text{tg} \delta_3} = \frac{B_2 \text{tg} \delta_2 + L_{c2}}{L_{c2} \text{tg} \delta_3} \\ \Rightarrow \delta_4 = \arctan \frac{L_{c2} \text{tg} \delta_3}{B_2 \text{tg} \delta_3 + L_{c2}} \quad \text{[rad]} \quad (9)$$

Thus, from the input parameter, the guide wheel rotation angle of 1 :  $\delta_1$  we can determine the angle of rotation of the remaining guide wheels:

$$\rightarrow \begin{cases} \delta_2 = \arctan \frac{L_{c1} \text{tg} \delta_1}{B_1 \text{tg} \delta_1 + L_{c1}} = \arctan \frac{L_{c1} \text{tg} \delta_1}{2l \text{tg} \delta_1 + L_{c1}} \\ \delta_3 = \arctan \left( \text{tg}(\delta_1) \cdot \frac{L_{c2}}{L_{c1}} \right) \\ \delta_4 = \arctan \frac{L_{c2} \text{tg} \delta_3}{B_2 \text{tg} \delta_3 + L_{c2}} = \arctan \frac{L_{c2} \text{tg} \delta_3}{2l \text{tg} \delta_3 + L_{c2}} \end{cases} \quad (10)$$



**Fig 3. Kinetic relation of the rotation angle of the guide wheels in the reverse direction**

$$L = L_{c1} + L_{c2} \quad (11)$$

$$\text{tg} \delta_1 = \frac{L_{c1}}{R}; \text{tg} \delta_2 = \frac{L_{c1}}{R+B_1}; \text{tg} \delta_3 = \frac{L_{c2}}{R}; \text{tg} \delta_4 = \frac{L_{c2}}{R+B_2} \quad (12)$$

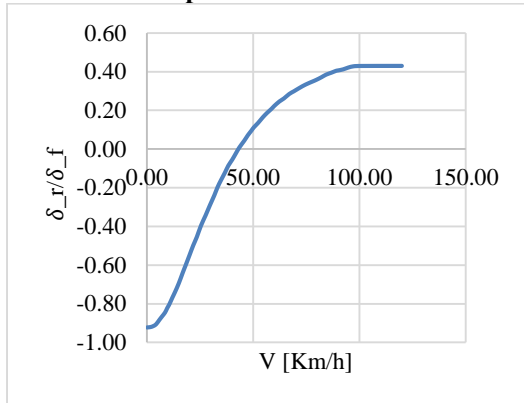
$$\text{Cotg} \delta_2 - \text{Cotg} \delta_1 = \frac{B_1}{L_{c1}} \quad (13)$$

$$\text{Cotg} \delta_4 - \text{Cotg} \delta_3 = \frac{B_2}{L_{c2}} \quad (14)$$

Thus, we can determine the wheelbase L through the parameters of the base width and the rotation angles of the guide wheels:

$$L = \frac{B_1}{\text{Cotg} \delta_2 - \text{Cotg} \delta_1} + \frac{B_2}{\text{Cotg} \delta_4 - \text{Cotg} \delta_3} \quad (15)$$

**2.3. The relationship between the rear guide wheel rotation angle relative to the front with the speed**



**Fig 4. The relationship between the rear guide wheel rotation angle relative to the front speed according to the speed [3]**

At velocity  $V= 42$  [Km/h]

$$\rightarrow \frac{\delta_r}{\delta_f} = 0$$

At velocity  $V= 32$  [Km/h]

$$\rightarrow \frac{\delta_r}{\delta_f} = -0.24 \rightarrow \delta_f = 30 \text{ [deg]}; \delta_r = -7.2 \text{ [deg]}$$

$$k=-0.18$$

At velocity  $V= 50$  [Km/h]

$$\rightarrow \frac{\delta_r}{\delta_f} = 0.11 \rightarrow \delta_f = 10 \text{ [deg]}; \delta_r = 1.1 \text{ [deg]}$$

$$k=0.125$$

**3. Investigating the dynamics of changing the direction of motion of a 4WS vehicle**

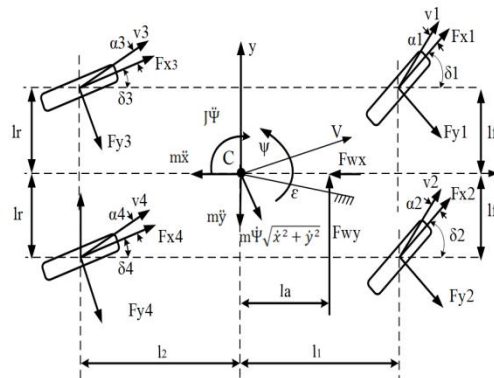
The surveyed vehicle is a reference vehicle with the following parameters [Table 1]:

**Table 1. Reference car model parameters**

N	Parameter	Value	Unit
1	The total mass of the car	$m = 1070$	[Kg]
2	Moment of inertia about a vertical axis passing through the car's center of gravity	$J = 2100$	[Kg.m <sup>2</sup> ]
3	Distance from center of gravity to front axle	$l_1=1,1$	[m]
4	Distance from center of gravity to rear axle	$l_2=1,3$	[m]
5	Front/rear wheel tracks	1400/1400	[Mm]
5	Wheel radius	$r_{bx}=0,32$	[m]
8	Wheel moment of inertia	$J_b = 3,6$	[Kg.m <sup>2</sup> ]
9	Air resistance coefficient	$K = 0,4$	[Ns <sup>2</sup> /m <sup>4</sup> ]
10	Area of the front bumper of the car	$F = 2$	[m <sup>2</sup> ]

**3.1. Building mathematical models**

The dynamic model of the rotational motion of a two-track car is shown above



**Fig 4. Two-track model of 4WS steering system [2]**

**Inside :**

$F_{x1}, F_{x2}, F_{x3}, F_{x4}$  : the component of the longitudinal reaction force, located at the center of the contact traces of the wheels with the road surface, with the direction lying on the centerline of the wheel.

$F_{y1}, F_{y2}, F_{y3}, F_{y4}$  : the horizontal reaction acting on the wheel, perpendicular to the longitudinal plane of the wheel

$\delta_1, \delta_2, \delta_3, \delta_4$  : the rotation angles of the guide wheel.

$\alpha_1, \alpha_2, \alpha_3, \alpha_4$  : the lateral roll angle of the wheels.

$$\alpha_1 = a \tan \frac{\dot{y}_1}{\dot{x}_1} - \delta_1 = a \tan \frac{\dot{y} + \dot{\psi} l_1}{\dot{x} - \dot{\psi} l_f} - \delta_1 \approx \frac{\dot{y} + \dot{\psi} l_1}{\dot{x} - \dot{\psi} l_f} - \delta_1 \quad (16)$$

$$\alpha_2 = a \tan \frac{\dot{y}_2}{\dot{x}_2} - \delta_2 = a \tan \frac{\dot{y} + \dot{\psi} l_1}{\dot{x} + \dot{\psi} l_f} - \delta_2 \approx \frac{\dot{y} + \dot{\psi} l_1}{\dot{x} + \dot{\psi} l_f} - \delta_2 \quad (17)$$

$$\alpha_3 = a \tan \frac{\dot{y}_3}{\dot{x}_3} - \delta_3 = a \tan \frac{\dot{y} - \dot{\psi} l_2}{\dot{x} - \dot{\psi} l_r} - \delta_3 \approx \frac{\dot{y} - \dot{\psi} l_2}{\dot{x} - \dot{\psi} l_r} - \delta_3 \quad (18)$$

$$\alpha_4 = a \tan \frac{\dot{y}_4}{\dot{x}_4} - \delta_4 = a \tan \frac{\dot{y} - \dot{\psi} l_2}{\dot{x} + \dot{\psi} l_r} - \delta_4 \approx \frac{\dot{y} - \dot{\psi} l_2}{\dot{x} + \dot{\psi} l_r} - \delta_4 \quad (19)$$

With the number 1 and 2 active wheels, we have a mathematical model to change the direction of motion of the 4WS car:

$$\begin{cases} m\ddot{x} = F_{x1} \cos \delta_1 + F_{x2} \cos \delta_2 + F_{y1} \sin \delta_1 + F_{y2} \sin \delta_2 + \\ F_{y3} \sin \delta_3 + F_{y4} \sin \delta_4 + m \cdot \dot{y} \cdot \dot{\psi} - F_{ax} \\ m\dot{y} = F_{x1} \sin \delta_1 + F_{x2} \sin \delta_2 - F_{y1} \cos \delta_1 - F_{y2} \cos \delta_2 - \\ F_{y3} \cos \delta_3 - F_{y4} \cos \delta_4 - m\dot{x} \cdot \dot{\psi} + F_{ay} \\ J\ddot{\psi} = l_f(-F_{x1} \cos \delta_1 - F_{y1} \sin \delta_1 + F_{x2} \cos \delta_2 + F_{y2} \sin \delta_2) + \\ l_r(F_{x1} \sin \delta_1) - m\dot{x} \cdot \dot{\psi} + F_{ay} \end{cases} \quad (20)$$

### 3.2. Building a block diagram to survey the movement of a car

Case 1: At velocity  $V = 32$  [Km/h]

$$\rightarrow \frac{\delta_r}{\delta_f} = -0.24$$

$\rightarrow \delta_f = 30$  [deg];  $\delta_r = -7.2$  [deg]  $k = -0.18$  and the rear tire lateral stiffness is smaller than the standard specification. Let the guide wheel No. 1 (front) rotate about the vertical pillar at an angle of 30 degrees [fig. 5]. The rear guide wheels are in a straight motion (non-adjustable) position. The trajectory of the car's movement according to the fixed coordinate system mounted on the road is shown in [Fig 6]. Cars turn around with a radius of about 5.6m.

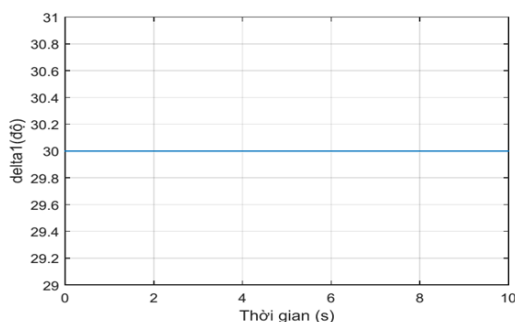


Fig 5. Guide wheel rotation angle when turning with constant radius

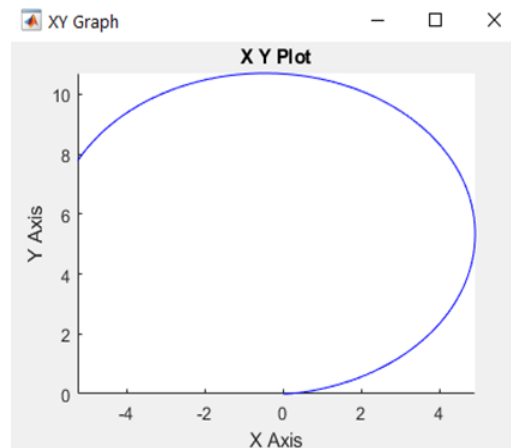


Fig 6. Car's motion trajectory when turning with constant radius on car 2WS

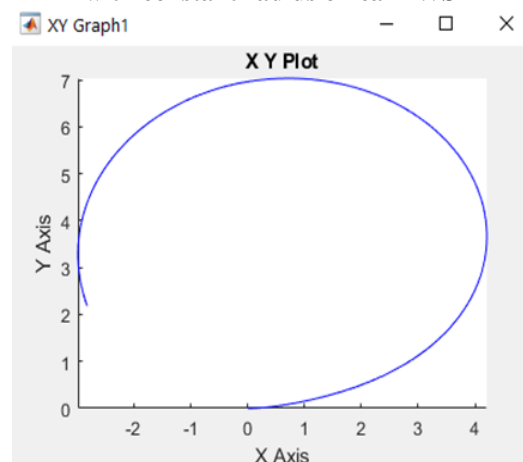


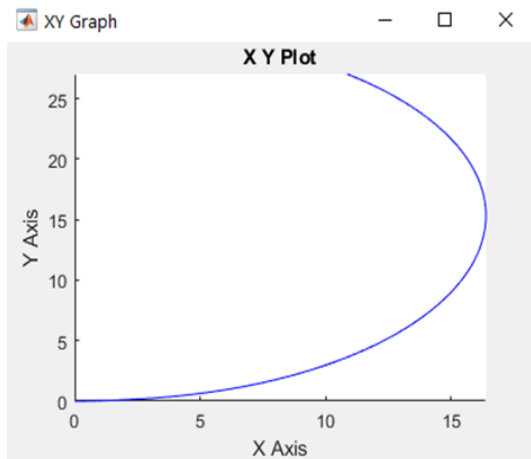
Fig 7. Car's motion trajectory when turning with a constant radius

When  $K=-0.18$  is set, the front and rear guide wheels are in the opposite direction and the turning radius of the car is reduced to 3.5 [m].

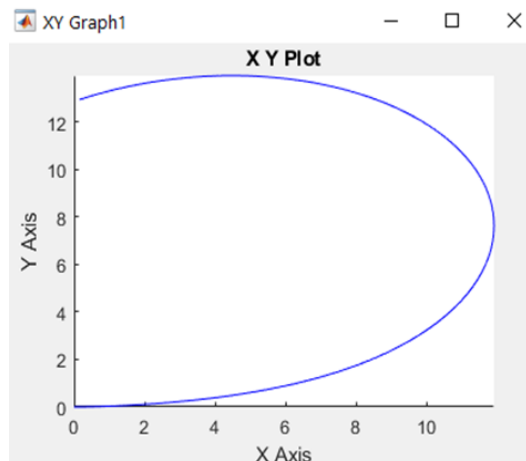
**Case 2: At velocity  $V= 50$  [Km/h]**

$\rightarrow \frac{\delta_r}{\delta_f} = 0.11$

$\rightarrow \delta_f = 10[\text{deg}]; \delta_r = 1.1[\text{deg}]$   $k=0.125$  and the rear tire lateral stiffness is smaller than the standard parameter [Fig 8], which is the vehicle's turning trajectory with the rear tire's lateral stiffness less than the standard parameter, the rear wheel is not controlled. Line 1 corresponds to the vehicle's trajectory with standard tire parameters. Line 2 corresponds to the vehicle's trajectory when the rear tire's horizontal stiffness decreases ( $C_{y3} = C_{y4} = 0,3C_y$ ).



**Fig 8. Car's motion trajectory when turning with constant radius on car 2WS**



**Fig 9. Vehicle trajectory when turning at a speed of 50 [km/h]**

When reducing the horizontal stiffness of the rear tire is less than the standard parameter ( $C_{y3} = C_{y4} = 0,3C_y$ ), The turning radius  $R \approx 7\text{m}$  is smaller than that of the standard tire parameter (overturning), causing instability in movement, the vehicle moves incorrectly compared to the trajectory.

**4. Conclude**

Based on the built mathematical model, the author uses MATLAB/Simulink software to simulate the rotational dynamics of a car with a set of parameters of a specific vehicle. Perform the simulation of the rotation trajectory with different cases when changing the lateral stiffness of the tire. The author has built the relationship between the rotation angle of the rear guide wheel compared to the front according to the speed through an available data map. The research is the basis for designing the controller of the 4WS steering system.

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