

# Restoration of Thermal Two-Dimensional Fields by Bicubic Splines

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**ABSTRACT:** A method for calculating the temperature field of a printed circuit board is presented, based on the approximation by a system of bicubic splines of the sets of measured temperatures at selected points of the printed circuit board. The method makes it possible to determine the temperature of a printed circuit board at any point and can be applied in calculating the temperature regimes of operation of printed circuit boards that are part of the coordinate-time support equipment. In the process of developing the designs of equipment for coordinate-time support (CVO), the tasks of calculating and analyzing the processes of heat release of units and blocks of CVO equipment constantly arise, on the basis of which the choice of design solutions is made in the design of systems. Significant assistance in the analysis of thermal processes in the designs of printed circuit boards (PCB) can be provided by specialized software that allows you to carry out the necessary calculations at the early stages of designing CVO equipment and is available to a user who does not have special training. To analyze the thermal regimes of the PCB of the CVO equipment, it is proposed to use the BetaSoft Board software from Dynamic Soft Analysis Inc (USA). The program has a developed mathematical apparatus, which makes it possible to carry out a full-fledged three-dimensional modeling of heat transfer phenomena on the PCB. Simulation accuracy in this case is about 10% compared to full-scale tests. However, with all the positive aspects of the BetaSoft Board software, it is difficult to determine the temperature value at a given point of the SP, since all calculations are displayed as temperature gradients in certain intervals, i.e. in fact, exact temperature values are only available at the boundaries of the temperature

zones. The method of approximating the temperature values of the PP by bicubic splines considered in the article makes it possible to eliminate this disadvantage and replace the continuous function of two variables with a combination of functions, each of which depends on one variable.

**KEYWORDS:** thermal field model, printed circuit board, temperature distribution, thermal power plane, approximation, coordinate-time support.

## I. INTRODUCTION

In the process of developing the designs of equipment for coordinate-time support (CTS), the tasks of calculating and analyzing the processes of heat release of units and blocks of CTS equipment constantly arise, on the basis of which the choice of design solutions is made in the design of systems. Significant assistance in the analysis of thermal processes in the designs of printed circuit boards (PCB) can be provided by specialized software that allows you to carry out the necessary calculations at the early stages of designing CTS equipment and is available to a user who does not have special training.

Calculation of thermal regimes of units and units of CTS equipment is based on the principle of electrothermal analogy. This principle lies in the fact that the transfer of thermal energy in structures is considered similar to the transfer of electricity in electrical circuits. In this case, the power of the heated zone  $P_{hz}$  acts as an analogue of the current strength; analogue of the potential difference - temperature difference (or overheating)  $\Delta T$  of the heated zone  $T_{hc}$  and ambient temperature  $T_{am}$ ; analogue of

electrical conductivity - thermal conductivity  $\delta$ . The use of such an analogy makes it possible to draw up thermal circuits and calculate them according to the basic rules of electrical engineering [1,6,8].

## II. PROBLEM STATEMENT

Usually, the thermal power released in the unit/block of the CTS equipment is set, the operating conditions ( $T_{am}$ ) are indicated, and the design parameters of the unit/block are known. Evaluation of the thermal regime of the block consists in the step-by-step determination of the overheating of the heated zones:  $\Delta T = Phz / \delta$ .

By changing the design parameters of heat pipelines and the conditions for removing heat from the outer boundaries of the unit/block of the CTS equipment, it is necessary to strive to reduce the overheating value  $\Delta T$  to the limit. The task is to ensure that, with a given design of the unit / block of the CTS equipment, the temperature of the heated zone does not exceed the limit value (for this specific element and specific circuit). In all cases, it is desirable to design and calculate the heat removal system so that  $\Delta T$  does not exceed 5-10 °C.

## III. SOFTWARE USED

To analyze the thermal regimes of the PCB of the CTS equipment proposed to use the BetaSoft Board software from Dynamic Soft

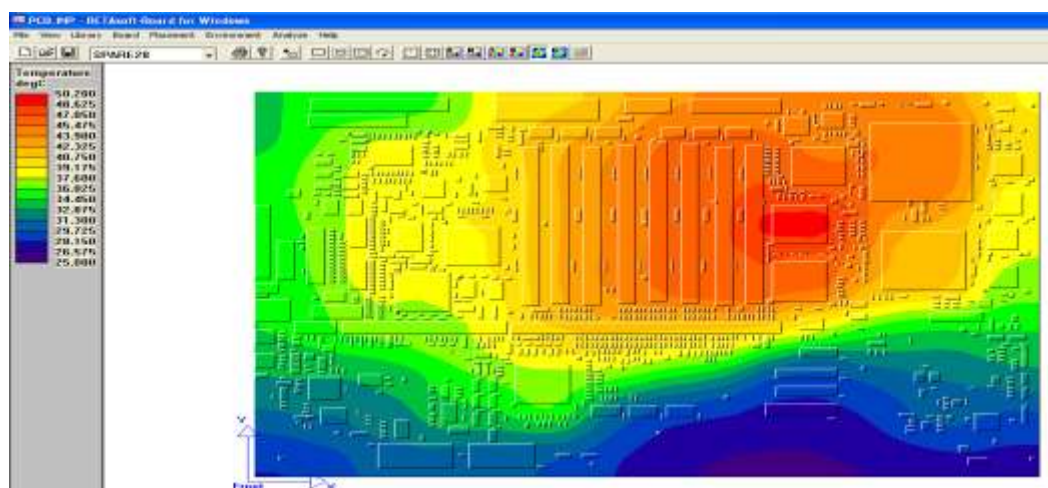
Analysis Inc (USA). The program has a developed mathematical apparatus that allows for a full-fledged three-dimensional modeling of heat transfer phenomena on the PCB.

In this case, the modeling accuracy is about 10% compared to full-scale tests [6, 7].

In addition, the BetaSoft-Board software has a special converter that provides automatic transfer of data on the design of the PCB and the electrical and radio products placed on its surface from the computer-aided design system (CAD) Expedition PCB.

When modeling the thermal regime of PCBs, the BetaSoft-Board software uses numerical methods based on finite difference methods with adaptive grids, which allow coarsening the mesh step on small PCB details that do not fall into the main, larger grid [6]. The advantage of this approach is the increased speed of analysis while maintaining high accuracy of calculations. The typical computation time for a PCB consisting of 100 components on a class 486 personal computer is about 30 seconds [6, 12]. The processes of heat transfer, heat conduction, convection and radiation are modeled.

However, with all the positive aspects of the BetaSoft Board software, it is difficult to determine the temperature value at a given point of the SP, since all calculations are displayed as temperature gradients in certain intervals, i.e. in fact, accurate temperature values are only available at the boundaries of temperature zones (Figure 1).



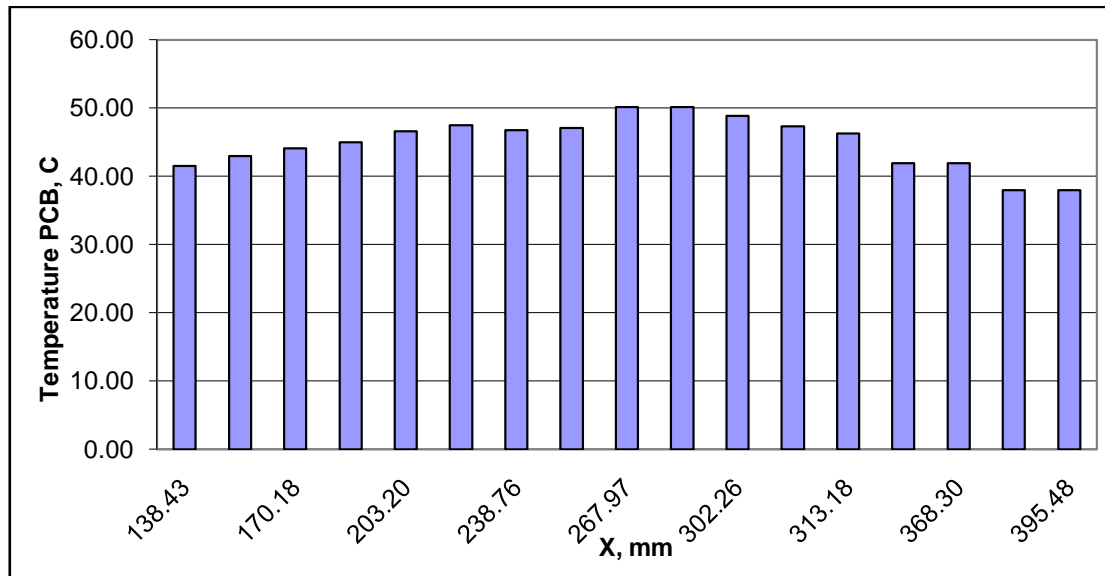


Figure 1. General view of the BCP and temperature values of the BCP at the boundaries of temperature zones.

#### IV. THERMAL FIELD SIMULATION METHOD BASED ON BICUBIC SPLINES.

The proposed method for approximating the temperature values of the BCP by splines makes it possible to eliminate this disadvantage and replace the complex continuous function of two variables with a combination of functions, each of which depends on one variable [2,3,4,5]. The use of local interpolation spline functions for constructing mathematical models in the problems of digital processing and restoration of various signals remains relevant [3,9,10,11,18,19]. The function  $S_n(f; x)$  is called the  $n$  power function of local interpolation provided that the following conditions are met:

- $S_n(f; x) \in H_n[x_i, x_{i+1}]$
- $S_n(x) \in C_n[a, b]$
- $S_n(x_i) \in f(x_i) \quad i = 0, n$

The defect of an interpolating spline function of degree  $n$  is the number  $v = n - m$ . Two-dimensional local interpolation spline functions can be built on the basis of one-dimensional local cubic interpolation spline functions, and the error estimate is also based on the errors of local cubic interpolation spline functions. The construction of a local parabolic spline is as follows.

To construct the domain  $D = [a, b] \times [c, d]$ , we divide these intervals into  $N$ , equal to the OX axis, and  $M$ , equal to the axis OY,  $\Delta = \Delta_x \times \Delta_y$

$$\Delta_x : a = x_0 < x_1 < \dots < x_N = b,$$

$$\Delta_y : c = y_0 < y_1 < \dots < y_M = d.$$

where  $h$  and  $l$  are chosen as follows

$$h = x_{i+1} - x_i, \quad i = 0, 1, \dots, N - 1; \quad l = y_{j+1} - y_j$$

$$j = 0, 1, \dots, M - 1.$$

Consider a grid:

$$\Delta^* = \Delta_x^* \times \Delta_y^*$$

$$\Delta_x^* : x_{-1} < x_0 < x_1 < \dots < x_N < x_{N+1},$$

$$\Delta_y^* : y_{-1} < y_0 < y_1 \dots < y_M < y_{M+1}$$

Then we have

$$D^* = [a - h, b + h] \times [c - 1, d + 1]$$

at the node

$$\Delta^* = \Delta_x^* \times \Delta_y^* \text{ -the values of the function are known, that is:}$$

$$f(x_i, y_j) = f_{ij}, \quad i = -1, 0, 1, \dots, N, N + 1$$

$$j = -1, 0, 1, \dots, M, M + 1$$

Based on the above values, a parabolic local interpolating spline [2,4,5,17] interpolating the functions  $f(x,y)$  in the domain  $D$  will be constructed. Since the parabolic spline interpolating  $f(x,y)$  is local, it is built on the basis of values of the function  $f_{ij}$  in the range

$$[x_i, x_{i+1}] \times [y_i, y_{i+1}]$$

$$(x_{i-1}, y_{i-1}), (x_{i-1}, y_j), (x_{i-1}, y_{j+1}), (x_{i-1}, y_{j+2}),$$

$$(x_i, y_{j-1}), (x_i, y_j), (x_i, y_{j+1}), (x_i, y_{j+2})$$

$$(x_{i+1}, y_{j-1}), (x_{i+1}, y_j), (x_{i+1}, y_{j+1}), (x_{i+1}, y_{j+2})$$

$$S_3(x_{i-1}, y) = (1-u)Z_j(x_{i-1}, y) + uZ_{j+1}(x_{i-1}, y) \quad (5)$$

$$(x_{i+2}, y_{j-1}), (x_{i+2}, y_j), (x_{i+2}, y_{j+1}), (x_{i+2}, y_{j+2})$$

$$S_3(x_{i+1}, y) = (1-u)Z_j(x_{i+1}, y) + uZ_{j+1}(x_{i+1}, y) \quad (6)$$

It should be noted that the spline constructed for a fixed value of one of the variables is a one-dimensional cubic local interpolation spline [2,3,4,5,16]. Here - is fixed, that is, the local interpolation cubic function  $x=x_i$  has the following form

$$S_3(x_i, y)$$

$$S_3(x_i, y) = (1-u)Z_j(x_i, y) + uZ_{j+1}(x_i, y) \quad (1)$$

Where

$$Z_j(x_i, y) = -\frac{1}{2}u(1-u)f_{i,j-1} + (1-u^2)f_{i,j} + \frac{1}{2}u(1+u)f_{i,j+1} \quad (2)$$

$$Z_{j+1}(x_i, y) = -\frac{1}{2}u(1-u)f_{i,j} + (1-u)f_{i,j+1} - \frac{1}{2}u(1+u)f_{i,j+2} \quad (3)$$

The parabolas  $(Z_j(x_{i,j}), Z_{j+1}(x_{i,j}))$  are as follows

$$(x_i, y_{j-1}), (x_i, y_j), (x_i, y_{j+1});$$

$$(x_i, y_j), (x_i, y_{j+1}), (x_i, y_{j+2});$$

$$u = \frac{y - y_j}{1}, l = y_{j+1} - y_j.$$

parabolas that pass through the node. Substituting (2) and (3) into (1), after some simplification, we get the following:

$$S_3(x_i, y) = -\frac{1}{2}u(1-u)^2 f_{i,j-1} + (1-u)(2 + 2u - 3u^2)f_{i,j} +$$

$$\frac{1}{2}u(1 + 4u - 3u^2)f_{i,j+1} - \frac{1}{2}u^2(1-u)f_{i,j+2} \quad (4)$$

$$j = \overline{0, M-1}, 0 \leq u \leq 1.$$

Based on the foregoing

$$x_i = x_{i-1}; x_{i+1}; x_{i+2};$$

In the fixed case, we get the following one-dimensional spline functions: after certain simplifications, we obtain the following local interpolation spline function:

$$S_{3,3}(x, y) = \varphi_1(t)[\varphi_1(u)f_{i-1,j-1} + \varphi_2(u)f_{i-1,j} + \varphi_3(u)f_{i-1,j+1} + \varphi_4(u)f_{i,j+2}] + \varphi_3(t)[\varphi_1(u)f_{i,j-1} + \varphi_2(u)f_{i,j} + \varphi_3(u)f_{i,j+1} + \varphi_4(u)f_{i,j+2}] + \varphi_3(t)[\varphi_1(u)f_{i,j+1} + \varphi_2(u)f_{i,j+2} + \varphi_3(u)f_{i,j+3} + \varphi_4(u)f_{i,j+4}] + \varphi_3(t)[\varphi_1(u)f_{i,j+2} + \varphi_2(u)f_{i,j+3} + \varphi_3(u)f_{i,j+4} + \varphi_4(u)f_{i,j+5}]$$

$$S_3(x_{i+2}, y) = (1-u)Z_j(x_{i+2}, y) + uZ_{j+1}(x_{i+2}, y) \quad (7)$$

$$S_3(xx-, y) = S_3(xi - y)S_3(xx + 1, y) \text{ and } S_3(xx + 2, y)$$

Based on the one-dimensional cubic splines obtained above, after some transformations, it is possible to construct two-dimensional interpolation splines of the following form [4]:

$$S_{3,3}(x, y) = -\frac{1}{2}t(1-t)^2 S_3(x_{i-1}, y) + \frac{1}{2}(1-t)S_3(x_i, y) + \frac{1}{2}t(1+4t-3t^2)S_3(x_{i+1}, y) - \frac{1}{2}t^2(1-t)S_3(x_{i+2}, y)$$

$$j = \overline{0, M-1}, 0 \leq u \leq 1.$$

$$t = \frac{x - x_i}{h}, u = \frac{y - y_j}{1},$$

$$h = x_{i+1} - x_i, l = y_{j+1} - y_j,$$

Substituting the values of the one-dimensional cubic splines constructed above  $S_3(x_{i-1}, y)$ ,  $S_3(x_i, y)$ ,  $S_3(x_{i+1}, y)$  and  $S_3(x_{i+2}, y)$  we get:

$$S_3(x_i, y) = -\frac{1}{2}t(1-t^2)[(1-u)Z_j(x_{i-1}, y) + uZ_{j+1}(x_{i-1}, y)] + \frac{1}{2}(1-t)$$

$$2 + 2t + 3t^2[(1-u)Z_j(x_i, y) + uZ_{j+1}(x_i, y)] + \frac{1}{2}t(1 + 4t - 3t^2)$$

$$[(1-u)Z_j(x_{i+1}, y) + uZ_{j+1}(x_{i+1}, y)] - \frac{1}{2}t^2(1-t)[(1-u)$$

$$\cdot Z_j(x_{i+2}, y) + uZ_{j+1}(x_{i+2}, y)] \quad (8)$$

$$\text{Where } i = \overline{0, N-1} \quad j = \overline{0, M-1},$$

$$0 \leq u \leq 1. 0 \leq u \leq 1. t = \frac{x - x_i}{h},$$

$$u = \frac{y - y_j}{1}, h = x_{i+1} - x_i, l = y_{j+1} - y_j,$$

$$+ \varphi_4(u)f_{i-1,j+2}] + \varphi_2(t)[\varphi_1(u)f_{i,j-1} + \varphi_2(u)f_{i,j} + \varphi_3(u)f_{i,j+1} + \varphi_4(u)f_{i,j+2}] + \varphi_3(t)[\varphi_1(u)f_{i,j+1} + \varphi_2(u)f_{i,j+2} + \varphi_3(u)f_{i,j+3} + \varphi_4(u)f_{i,j+4}] + \varphi_3(t)[\varphi_1(u)f_{i,j+2} + \varphi_2(u)f_{i,j+3} + \varphi_3(u)f_{i,j+4} + \varphi_4(u)f_{i,j+5}]$$

$$\begin{aligned}
 & + \varphi_3(u) f_{i+1,j+1} + \varphi_4(u) f_{i+1,j+2} ] + \varphi_4(t) [ \varphi_1(u) f_{i+2,j} + \varphi_2(u) f_{i+2,j+1} + \varphi_3(u) f_{i+2,j+2} ] \\
 & + \varphi_3(u) f_{i+2,j+1} + \varphi_4(u) f_{i+2,j+2} ]
 \end{aligned}$$

(9)

Where  $\varphi_1(t) = -\frac{1}{2}t(1-t)^2$   
 $\varphi_2(t) = \frac{1}{2}t(1-t)(2+2t-3t^2)$

$$\varphi_4(t) = -\frac{1}{2}t^2(1-t)$$

The resulting mathematical model can be used to restore various thermal, seismic, geophysical fields of complex structure [3,13,14,15]. The table shows the measured temperatures of the printed circuit board at given points. The graph of the initial thermal field according to the table is shown in Fig.2. The results of the thermal field reconstruction using the mathematical model (9) built on the basis of the bicubic spline are shown in Fig.3.

Table  
 Measured PCB temperatures at given points

x, mm	T <sup>0</sup> C,x	ST <sup>0</sup> ,x	y, mm	T <sup>0</sup> C,y	ST <sup>0</sup> ,y
48.0	37.60	37.60	90.9	32.88	32.86
110.0	39.17	39.18	94.8	34.45	34.48
141.3	40.75	40.75	99.5	36.03	36.00
158.5	42.32	42.32	104.3	37.60	37.46
181.4	43.90	43.90	111.0	39.18	39.30
204.4	45.47	45.48	118.2	40.75	40.82
246.5	47.05	47.06	123.4	42.33	42.32
262.7	48.63	48.61	128.7	43.90	42.74
298.6	48.63	48.59	135.9	45.48	45.50
309.11	47.05	47.08	162.9	47.05	47.12
317.2	45.47	45.49	174.1	48.63	48.72
328.7	43.90	43.91	191.8	48.63	48.68
356.4	42.32	42.32	206.6	47.05	46.96

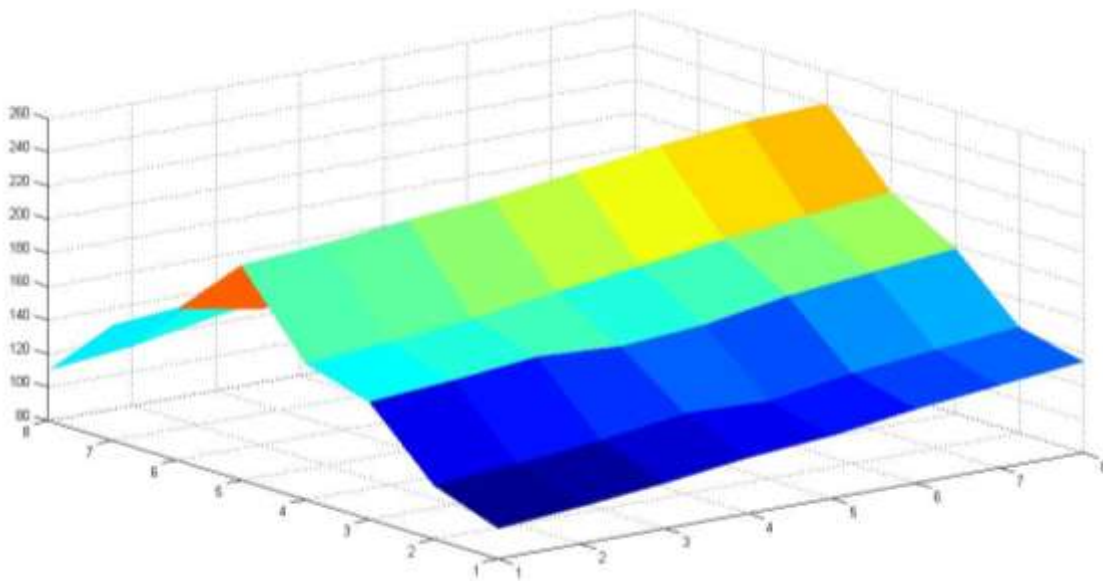


Fig.2. Graph of the initial thermal field according to table 1.

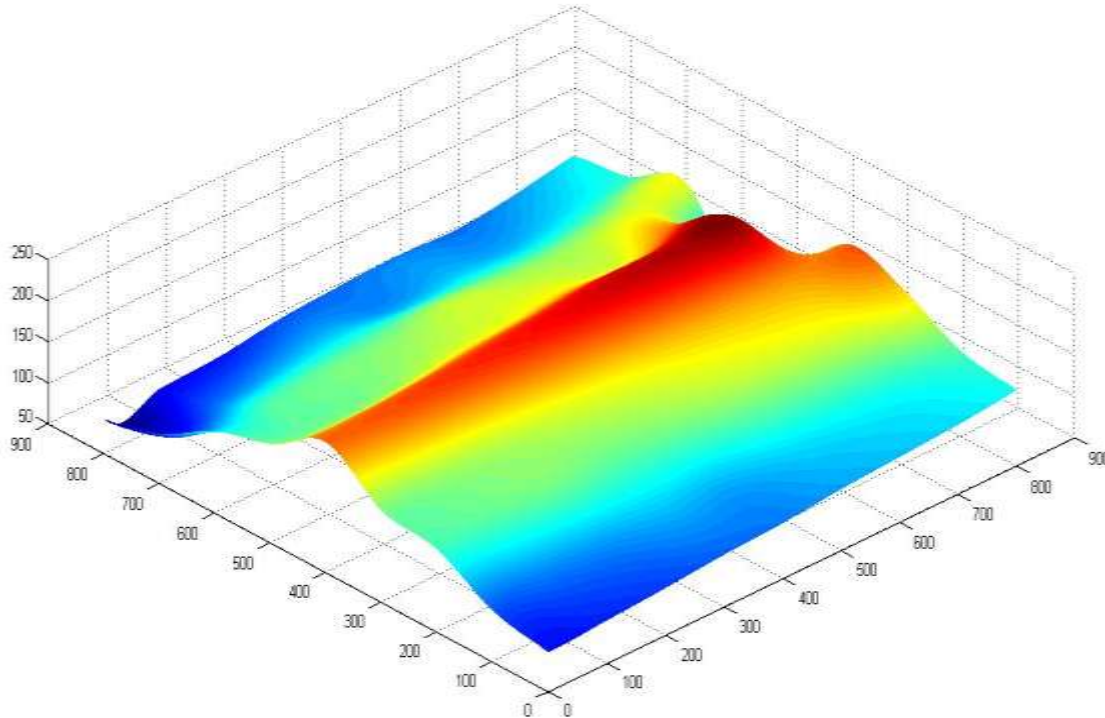


Fig.3. The results of the thermal field reconstruction using the mathematical model (9)

## V. CONCLUSION

Thus, splines as a class of piecewise functions, due to a number of advantages over other interpolation methods, are increasingly used in the development of algorithmic and software tools for analyzing and recovering signals, expanding the scope of traditional approaches to modeling complex fields. The advantages of splines in problems of calculating functions of many variables are indisputable. They can be used to determine anomalous changes in thermal, electromagnetic, gravitational fields, anomalous disturbances in the ionosphere, seismic noise, various acoustic vibrations, and others.

## REFERENCES

- [1]. Arnold V.I. On the representability of functions of two variables in the form  $\square \square \square (x) \square \square (y) \square$  // Usp. Mat. Sciences. 1957. Issue 2. pp.119-121.
- [2]. Zavyalov Yu.S., Kvasov B.I., Miroshnichenko I.L. Methods of spline - functions. - M.: Nauka, 1980. 352 p.
- [3]. Zaynidinov H.N. // Splines in problems of digital signal processing./ Monograph. Academy of Public Administration under the President of the Republic of Uzbekistan. - T.: "Fan vatehnologiya", 2015. -208 p.
- [4]. Zaynidinov H.N., Bakhrarov S.A. Theory of splines // Monograph -T.: "Aloqachi", 2019.- 174 p.
- [5]. Svin'in S.F. Basic splines in the theory of signal readings. St. Petersburg: Nauka, 2003. -118p.
- [6]. Steshenko V.B. Software for the analysis of thermal conditions of printed circuit boards BetaSoft Board - Circuitry, No. 3, 2001.
- [7]. Kimberly Bobish CAE for thermal management of aerospace electronic boards using the BetaSoft program - NASA. Lewis Research Center, The Sixth Annual Thermal and Fluids Analysis Workshop, Jan 1995 - p 133-140.
- [8]. Mathematics and CAD. T.1 / Shenin P. et al. M.: Mir. 1988.
- [9]. Alberg J., Nilson E., Walsh J. Theory of splines and its applications. M.: Mir. 1972.
- [10]. Tuzov A.D. Smoothing functions specified by tables // Computational systems. Issue.68. Novosibirsk. 1976. S.61-66.
- [11]. Shura-Bura M.R. Approximation of functions of many variables by functions, each of which depends on one variable /

- Computational mathematics: Sat. articles.  
Issue 2. M. 1957. S.3-19.
- [12]. Z. Zhengyu, "Digital processing of the polarization state of geophysical ULF signals," *Journal of Electronics (China)*, 11(3), 1994, doi:10.1007/BF02684833.
- [13]. A. Kroizer, Y.C. Eldar, T. Routtenberg, "Modeling and recovery of graph signals and difference-based signals," in *GlobalSIP 2019 - 7th IEEE Global Conference on Signal and Information Processing, Proceedings*, 2019, doi:10.1109/GlobalSIP45357.2019.8969536.
- [14]. A.K. Takahata, E.Z. Nadalin, R. Ferrari, L.T. Duarte, R. Suyama, R.R. Lopes, J.M.T. Romano, M. Tygel, "Unsupervised processing of geophysical signals: A review of some key aspects of blind deconvolution and blind source separation," *IEEE Signal Processing Magazine*, 29(4), 2012, doi:10.1109/MSP.2012.2189999.
- [15]. X.H. Liu, Z.Y. Zhao, S.G. Xie, J.H. Liu, "Time-frequency analysis of geophysical signals based on Cohen distributions," *DianboKexueXuebao/Chinese Journal of Radio Science*, 16(3), 2001.
- [16]. B.H. Jansen, "ANALYSIS OF BIOMEDICAL SIGNALS BY MEANS OF LINEAR MODELING.," *Critical Reviews in Biomedical Engineering*, 12(4), 1985.
- [17]. D. Singh, M. Singh, Z. Hakimjon, "Parabolic Splines based One-Dimensional Polynomial," 2019, doi:10.1007/978-981-13-2239-6\_1.
- [18]. R.O. Parker, "INTRODUCTION TO DIGITAL SIGNAL PROCESSING.," *J Eng ComputAppl*, 2(3), 1988, doi:10.1201/9781439817391-5.
- [19]. K. Daoudi, "Iterated Function Systems and Some Generalizations: Local Regularity Analysis and Multifractal Modeling of Signals," 2010.