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Research on Some Inverse Scheduling Problems

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ABSTRACT: In this paper, we summarize some results about the inverse scheduling problem $1|INV|\sum_{j=1}^{n} w_j C_j$ of the total weighted completion time problem on single machines $1||\sum_{j=1}^{n} w_j C_j$ and the inverse scheduling problem $Pm|INV|\sum_{j=1}^{n} C_j$ of the total completion time objective on parallel machines $Pm||\sum_{j=1}^{n} C_j$ in which the processing times $p=(p_1, p_2,..., p_n)^T$ are minimally adjusted, so that the given schedule is satisfying the necessary conditions and sufficient conditions for the scheduling problem $1||\sum_{j=1}^{n} w_j C_j$ and $Pm||\sum_{j=1}^{n} w_j C_j$ and becomes optimal with respect to $\overline{p}=(\overline{p}_1, \overline{p}_2,..., \overline{p}_n)^T$. We have obtained the mathematical programming formulations for this inverse scheduling problem with different norms and provided efficient solution algorithms.

KEYWORDS: Scheduling,Inverse Problem, Completion Time, Parallel Machine, Single Machine.

I. INTRODUCTION

In the recent past and in the recent year, many authors studied the inverse optimisation in schedulinh refers to the situation. There are a large number of article on inverse optimisation in schedulinh refers to the situation. Lun and Cariou [4]; Lun et al. [5], derived the processing times or the weights of the n jobs can be adjusted depending on the deployment of such resources as quay cranes to load/discharge containers on/from the ship and trucks to transport containers between the quayside and the container yard, so that the scheduling criterion (e.g., the tatal weighted completion time, which is summary measure of the waiting times of the jobs or the inventory level in the shop) is minimised with respect to the adjusted processing times or weights. However, the resulting value of the scheduing criterion may be higher than the original value of the scheduling criterion, wich is undesirable. Therefore we impose in this paper the constraint that the resulting value of the scheduling criterion based on the adjuted parameters should not be greater than the value of the scheduling criterion based on the original parameters.

II. THEINVERSESCHEDULINGPROBLEMOFTHETOTALCOMPLETIONTIMEOBJECTIVEONIDENTICALPARALLELMACHINES

In the forward scheduling problem $Pm||\sum_{j=1}^{n} C_j$, consider an arbitrary n-jobs $\{J_1, J_2,..., J_n\}$ should be processed by m parallel machines $\{M_1, M_2,..., M_m\}$. There are no precedence constraints between the jobs. Each job $J_j\,(j=1,2,...,n)$ has processing time $p_j\,(j=1,2,...,n)$. All jobs are available at time zero. For any schedules, assume that on machine $M_i\,(i=1,2,...,m), n_i$ jobs $\{J_{i,1}, J_{i,2},..., J_{i,n_1}\}$ are consecutively processed. So on machine $M_i\left(i=1,2,...,m\right)$, the completion time of job s is $C_{i,s}$ and the total completion time will be:

$$\sum_{s=1}^{n_i} C_{i,s} = \sum_{s=1}^{n_i}\sum_{l=1}^{s} p_{i,l} = \sum_{s=1}^{n_i} sp_{i,n_i-s+1}.$$

The total completion time on m machines $\sum_{j=1}^{n} C_j$ will be:

$$\sum_{j=1}^{n} C_j = \sum_{i=1}^{m}\sum_{s=1}^{n_i} C_{i,s} = \sum_{i=1}^{m}\sum_{s=1}^{n_i} sp_{i,n_i-s+1}.$$

As we know it is well-known Hongtruong Truong et al. [2] proved following the result:

As schedule $\sigma=\left(J_1,J_2,...,J_n\right)$ is optimal for problem $Pm\|\sum_{j=1}^{n} C_j$ if and only if for any given $S_a,S_b\left(a,b\in\{1,2,...,k\},a<b\right)$, there are $S_a \prec S_b$ and $p_i \le p_j$ for any $J_i\in S_a, J_j\in S_b$, where

$$S_1=\underbrace{\left\{J_1,J_2,...,J_h\right\}}_{h\ jobs},$$

$$S_2=\underbrace{\left\{J_{h+1},J_{h+2},...,J_{h+m}\right\}}_{m\ jobs}$$

....

$$S_{k+1}=\underbrace{\left\{J_{(k-1)m+h+1},J_{(k-1)m+h+2},...,J_{km+h}\right\}}_{m\ jobs}$$

In the inverse scheduling problem $Pm\,|\,INV\,|\sum_{j=1}^{n} C_j$, given a feasible schedule σ of the scheduling problem $Pm\,|\,INV\,|\sum_{j=1}^{n} C_j$, without loss of generality we assumethat $\sigma=\left(J_1,J_2,...,J_n\right)$, then the total completion time on m machines $\sum_{j=1}^{n} C_j$ will be:

$$\begin{aligned}\sum_{j=1}^{n} C_j &= \sum_{i=1}^{m}\sum_{s=1}^{n_i} sp_{i,n_i-s+1}\\ &= \sum_{j=(k-1)m+h+1}^{km+h} p_j + 2\left(\sum_{j=(k-2)m+h+1}^{(k-1)m+h} p_j\right)+...\\ &\quad + k\sum_{j=h+1}^{m+h} p_j + (k+1)\sum_{j=1}^{h} p_j\end{aligned}$$

where, $n=km+h, k\in\{1,2,...\}, h\in\{0,1,2,...,m-1\}$.

The problem $Pm\,|\,INV\,|\sum_{j=1}^{n} C_j$ is solved by determining the minimum total adjustable perturbation to the processing time $p=\left(p_1,p_2,...,p_n\right)^T$ to become $\overline{p}=\left(\overline{p}_1,\overline{p}_2,...,\overline{p}_n\right)^T$, so that the given schedule σ satisfies the necessary and sufficient conditions for optimality of the problem $Pm\|\sum_{j=1}^{n}C_j$ and becomes optimal with respect to $\overline{p}=\left(\overline{p}_1,\overline{p}_2,...,\overline{p}_n\right)^T$. Thus, we can formulate the scheduling problem $Pm\,|\,INV\,|\sum_{j=1}^{n}C_j$ as a mathematical programming problem:

$$\begin{aligned}
&\min \left\|\overline{p}-p\right\| \\
&s.t.\ \ \overline{p}_i\le\overline{p}_j \quad for\ any\ \ J_i\in S_l, \\
&\qquad J_i\in S_{l+1}, S_l\prec S_{l+1}\left(l=1,2,...,k\right) \quad (1) \\
&\quad \overline{p}_j\ge 0,\left(j=1,2,...,n\right).
\end{aligned}$$

where $\overline{p}_j$ is the new minimally perturbed processing time of job $j\left(j=1,2,...,n\right)$.

For above inverse schedule problem, we have different models under three types of norms: l_1- norm, l_2- norm, $l_\infty-$norm.

1. The inverse problem $Pm\,|\,INV\,|\sum_{j=1}^{n}C_j$ under l_2- norm

For l_2- norm, the formula (1) can be written as

$$\begin{aligned}
&\min\ \frac{1}{2}\sum_{j=1}^{n}\left(\overline{p}_j-p_j\right)^2 \\
&s.t.\quad \overline{p}_i\le\overline{p}_j \quad for\ any\ \ J_i\in S_l, \\
&\qquad\quad J_i\in S_{l+1}, S_l\prec S_{l+1}\left(l=1,2,...,k\right) \quad (2) \\
&\qquad \overline{p}_j\ge 0,\left(j=1,2,...,n\right).
\end{aligned}$$

The problem (2) is equivalentent to

$$\begin{aligned}
&\min\ \ f\left(\overline{p}\right)=\frac{1}{2}\left(\overline{p}\right)^T\overline{p}-p^T\overline{p}+\frac{1}{2}p^Tp \\
&s.t.\quad A\overline{p}\ge 0 \qquad\qquad\qquad\qquad (3) \\
&\qquad \overline{p}\ge 0,\left(j=1,2,...,n\right).
\end{aligned}$$

Where

$$A=\begin{bmatrix}M\\N\end{bmatrix}\in R^{\left((k-1)m^2+mh\right)\times n},$$

$$\begin{aligned}M=(&a_{1,1},a_{1,2},...,a_{1,m},a_{2,2},...,a_{2,m},a_{h,1},\\ &a_{h,2},...,a_{h,m})\in R^{(hm)\times n},\end{aligned}$$

$$N=(b_{h+1,h+m+1},\dots,b_{h+1,h+2m},\dots,b_{h+m,h+m+1},\dots,\\ b_{h+m,h+2m},\dots,b_{h+(k-1)m,h+m+1},\dots,\\ b_{h+(k-1)m,h+km})^T\in R^{((k-1)m^2)\times n},$$

$$b_{h+(i-1)m+x,h+im+y}=(0,\dots,0,\underbrace{-1}_{(h+(i-1)m+x)-th},\\ 0,\dots,0,\underbrace{1}_{(h+im+y)-th},\\ 0,\dots,0)^T\in R^n,\\ i=1,2,\dots,(k-1)\\ and\ \ x,y=1,2,\dots,m.$$

$$a_{x,y}=(0,\dots,0,\underbrace{-1}_{x-th},0,\dots,0,\underbrace{1}_{(h+y)-th},\\ 0,\dots,0)^T\in R^{n\times 1},\\ x=1,2,\dots,h\ \ and\ \ y=1,2,\dots,m,$$

Since $f\left(\overline{p}\right)$ is convex function and $D=\left\{\overline{p}\mid A\overline{p}\ge 0,\overline{p}\ge 0\right\}$ is convex set, the problem (3) is convex quadratic programming. So, its Kuhn-Tucker conditions (4) is the necessary and sufficient conditions for the optimal formula (3).

$$\begin{cases}\overline{p}-p-A^T\lambda-\mu=0,\\ A\overline{p}\ge 0,\\ \lambda^T\left(A\overline{p}\right)=0, & (4)\\ \mu^T\overline{p}=0,\\ \overline{p},\lambda,\mu\ge 0.\end{cases}$$

in which $\lambda\in R^{(k-1)m^2+mh\times 1},\mu\in R^{n\times 1}$.

By Wolfe algorithm of quadraticprogramming (D. Goldfar and A. Idnani [1]), we can easily solve of above Kuhn-Tucker conditions. Thus we can obtain the optimal solution of problem (2).

2. The inverse problem $Pm\mid INV\mid\sum_{j=1}^{n}C_j$ under l_1- norm

For l_1-norm, the problem (1) can be written as follows:

$$\begin{aligned}\min\ &\sum_{j=1}^{n}\left|\overline{p}_j-p_j\right|\\ s.t.\ \ &\overline{p}_i\le\overline{p}_j\ \ for\ any\ \ J_i\in S_l,\\ &J_i\in S_{l+1},S_l\prec S_{l+1}\left(l=1,2,\dots,k\right)\quad(5)\\ &\overline{p}_j\ge 0,\left(j=1,2,\dots,n\right).\end{aligned}$$

From (5) is a non-linear programming problem.
Let

$$\begin{cases} \alpha_j = \frac{1}{2}\left[\left|\overline{p}_j - p_j\right| + \left(\overline{p}_j - p_j\right)\right] \\ \beta_j = \frac{1}{2}\left[\left|\overline{p}_j - p_j\right| - \left(\overline{p}_j - p_j\right)\right] . \quad (6) \\ \qquad (j = 1,2,...,n) \end{cases}$$

By (6), we have

$$\begin{cases} \left|\overline{p}_j - p_j\right| = \alpha_j + \beta_j \\ \overline{p}_j = \alpha_j - \beta_j + p_j \quad (j = 1,2,...,n). \\ \alpha_j \geq 0 \\ \beta_j \geq 0 \end{cases}$$

Thus problem (5) is converted to the linear program as follows:

$$\begin{aligned} \min \quad & \sum_{j=1}^{n}\left(\alpha_j + \beta_j\right) \\ s.t. \quad & \alpha_i - \beta_i + p_i \leq \alpha_j - \beta_j + p_j \\ & \quad for\ any\ J_i \in S_l, J_i \in S_{l+1}, \\ & \quad\quad S_l \prec S_{l+1}\left(l = 1,2,...,k\right) \quad (7) \\ & \alpha_j - \beta_j + p_j \geq 0 \ \left(j = 1,2,...,n\right) \\ & \alpha_j, \beta_j \geq 0 \ \left(j = 1,2,...,n\right) \end{aligned}$$

According to linear programming (7) we can obtain optimal solution α_j and $\beta_j\left(j = 1,2,...,n\right)$. By $\overline{p}_j = \alpha_j - \beta_j + p_j$, , we find $\overline{p}_j\left(j = 1,2,...,n\right)$.

3. The inverse problem $Pm \mid INV \mid \sum_{j=1}^{n} C_j$ under $l_\infty -$ norm

For $l_\infty -$ norm, the mathematical program (1) of the inverse scheduling problem is

$$\begin{aligned} \min \ & \max_{1\leq j\leq n}\left|\overline{p}_j - p_j\right| \\ s.t. \quad & \overline{p}_i \leq \overline{p}_j \quad for\ any\ J_i \in S_l, \\ & \quad J_i \in S_{l+1}, S_l \prec S_{l+1}\left(l = 1,2,...,k\right) \quad (8) \\ & \overline{p}_j \geq 0, \left(j = 1,2,...,n\right). \end{aligned}$$

The program form (8) is also the non-linear programming, and is rewritten into

$$\begin{aligned} \min \ & \theta \\ s.t. \quad & \left|\overline{p}_j - p_j\right| \leq \theta \ \left(j = 1,2,...,n\right) \\ & \overline{p}_i \leq \overline{p}_j \quad for\ any\ J_i \in S_l, \\ & \quad J_i \in S_{l+1}, S_l \prec S_{l+1}\left(l = 1,2,...,k\right) \quad (9) \\ & \overline{p}_j \geq 0, \left(j = 1,2,...,n\right). \end{aligned}$$

By similarly transforms

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$$\begin{cases} \alpha_j = \frac{1}{2}\left[\left|\bar{p}_j - p_j\right| + \left(\bar{p}_j - p_j\right)\right] \\ \beta_j = \frac{1}{2}\left[\left|\bar{p}_j - p_j\right| - \left(\bar{p}_j - p_j\right)\right] \end{cases} (j = 1, 2, ..., n).$$

Problem (9) is converted to the form of linear programming as follows

$$\begin{aligned} &\min \quad \theta \\ &s.t. \quad \alpha_j + \beta_j \le \theta \quad (j = 1, 2, ..., n) \\ &\qquad \alpha_i - \beta_i + p_i \le \alpha_j - \beta_j + p_j \\ &\qquad\quad for\ any\ \ J_i \in S_l, \\ &\qquad\quad J_i \in S_{l+1}, S_l \prec S_{l+1} (l = 1, 2, ..., k) \qquad (10) \\ &\qquad \alpha_j - \beta_j + p_j \ge 0 \ (j = 1, 2, ..., n) \\ &\qquad \alpha_j, \beta_j \ge 0 \ (j = 1, 2, ..., n) \end{aligned}$$

Similarly, we can easily solve above linear programming. Thus, we can find $\bar{p}_j$ from the formula $\bar{p}_j = \alpha_j - \beta_j + p_j \ \ (j = 1, 2, ..., n)$.

III. CONCLUSION

In this paper, we have summarized some research results on the inverse scheduling problem $1 | INV | \sum_{j=1}^{n} w_j C_j$ and the inverse scheduling problem $Pm | INV | \sum_{j=1}^{n} C_j$ in which the processing times $p = (p_1, p_2, ..., p_n)^T$ are minimally adjusted, so that the given schedule σ is satisfying the necessary and sufficient conditions for optimality of the scheduling problem $1 || \sum_{j=1}^{n} C_j$ and $Pm || \sum_{j=1}^{n} C_j$ and becomes optimal with respect to $\bar{p} = (\bar{p}_1, \bar{p}_2, ..., \bar{p}_n)^T$. We have also produced their mathematical programming formulations and developed efficient solution algorithms, respectively.

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