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Research on (2+1) Dimensional BKK Equation

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ABSTRACT: In this paper, we derive exact traveling wave solutions of (2+1) dimensional BKK equation by a presented method.The method appears to be efficient in seeking exact solutionsof nonlinear equations.

KEYWORDS:(G'/G)-expansion method, traveling wave solutions,exact solution, evolution equation, nonlinear D-S equation.

I. INTRODUCTION

In scientific research, seeking the exact solutions of nonlinearequations is a hot topic. Many approaches have beenpresented so far [1-6]. In [7], Mingliang Wang propo-seda new method called (G'/G)-expansion method. The mainmerits of the (G'/G)-expansion method over the other methodsare that it gives more general solutions with some freeparameters and it handles NLEEs in a direct manner withno requirement for initial/boundary condition or initial trialfunction at the outset. So the application of the (G'/G)-expansion method attracts many author's attention.Our aim in this paper is to present an application ofthe (G'/G)-exp-ansion method to (2+1) dimensional BKK equation.

II. DESCRIPTION OF THE(G'/G)-EXPANSION METHOD

In this section we will describe the (G'/G)-expansion methodfor finding out the traveling wave solutions of no-nlinear evolution equations.

Suppose that a nonlinear equation, say in three indep-endentvariables x, y and t , is given by

$$P(u,u_{t},u_x,u_y,u_{tt},u_{xt},u_{yt},u_{xx},u_{yy}......)=0$$

(2.1)

where u = u(x, y, t) is an unknown function,P isa polyno-mial in u = u(x, y, t) and its various partialderivatives, in which the highest order derivatives andnonlinear terms are involved. In the following we give themain steps of the (G'/G)-expansion method.

Step 1. Combining the independent variables x, y and tinto one variable $\xi=\xi(x,y,t)$, we suppose that

$$u(x,y,t)=u(\xi),\xi=\xi(x,y,t) \quad (2.2)$$

the travelling wave variable (2.2) permits us reducing Eq.(2.1) to an ODE for $u=u(\xi)$

$$P(u,u',u'',......)=0$$

(2.3)

Step 2. Suppose that the solution of (2.3) can be expre-ssedby a polynomial in (G'/G) as follows:

$$u(\xi)=\alpha_m(\tfrac{G'}{G})^m+...... \quad (2.4)$$

where $G=G(\xi)$ satisfies the second order LODE in the form

$$G''+\lambda G'+\mu G=0 \quad (2.5)$$

$\alpha_m,...\lambda$ and μ are constants to be determined later, $\alpha_m\neq 0$ .The unwritten part in (2.4) is also a polynomial in $(\frac{G'}{G})$, the degree of which is generally equal to or less than m−1.The positive integer m can be determined by consider-ing thehomogeneous balance between the highest order de-rivativesand nonlinear terms appearing in (2.3).

Step 3. Substituting (2.4) into (2.3) and using secondorder LODE (2.5), collecting all terms with the sameorder of $(\frac{G'}{G})$ together, the left-hand side of Eq. (2.3)is converted into another polynomial in $(\frac{G'}{G})$. Equatingeach coefficient of this polynomial to zero, yields a set ofalgebraic equation-ns for $\alpha_m,...\lambda$ and μ.

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Step 4. Assuming that the constants $\alpha_m,...\lambda$ and μ

Canbe obtained by solving the algebraic equations in Step 3,

since the general solutions of the second order LODE (2.5)have been well known for us, substituting $\alpha_m,\cdots$ and thegeneral solutions of Eq. (2.5) into (2.4) we can obtain thetraveling wave solutions of the nonlinear evolution equation(2.1).

In the subsequent sections we will illustrate the propo-sedmethod in detail by applying it to a nonlinear evolutionequation.

III. APPLICATION OF (G'/G)-EXPANSION METHOD FOR(2+1) DIMENSIONAL BKK EQUATION

In this section, we will consider the following (2+1) dimensional BKK equation:

$$u_{ty} - u_{xxy} + 2(uu_x)_y + 2v_{xx} = 0 \quad (3.1)$$

$$v_t + v_{xxx} + 2uv_x = 0 \quad (3.2)$$

Supposing that

$$\xi = kx + ly + st \quad (3.3)$$

By (3.3), (3.1) and (3.2) are converted into ODEs

$$slu'' - k^2lu''' + 2kl(uu')' + 2k^2v'' = 0 \quad (3.4)$$

$$sv' + k^2v'' + 2k(uv)' = 0 \quad (3.5)$$

Integrating (3.4) and (3.5) once, we have

$$slu' - k^2lu'' + 2kluu' + 2k^2v' = 0 \quad (3.6)$$

$$sv + k^2v' + 2kuv = 0 \quad (3.7)$$

Suppose that the solution of (3.6) and (3.7) can be expressed by apolynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i (\tfrac{G'}{G})^i \quad (3.8)$$

$$v(\xi) = \sum_{i=0}^{n} b_i (\tfrac{G'}{G})^i \quad (3.9)$$

where a_i, b_i are constants, $G = G(\xi)$ satisfies the second orderLODE in the form:

$$G'' + \lambda G' + \mu G = 0 \quad (3.10)$$

where λ and μ are constants.

Balancing the order of uu' and v' in Eq.(3.6),the order of v'' and uv in Eq.(3.7), we can obtain $2m+1 = n+1, n+1 = m+n \Rightarrow m=1, n=2$.

So Eq.(3.8) and (3.9) can berewritten as

$$u(\xi) = a_1(\tfrac{G'}{G})^1 + a_0, a_1 \neq 0 \quad (3.11)$$

$$v(\xi) = b_2(\tfrac{G'}{G})^2 + b_1(\tfrac{G'}{G})^1 + b_0, b_2 \neq 0 \quad (3.12)$$

a_1, a_0, b_2, b_1, b_0 are constants to be determined later.

Substituting (3.11) and (3.12) into (3.6) and (3.7) and collecting all the termswith the same power of $(\frac{G'}{G})$ toge-ther and equating eachcoefficient to zero, yields a set of simultaneous algebraicequations as follows:

For Eq.(3.6):

$$(\frac{G'}{G})^0 : -k^2la_1\lambda\mu - lsa_1\mu - 2k^2b_1\mu - g_1 - 2kla_1a_0\mu = 0$$

$$(\frac{G'}{G})^1 : -lsa_1\lambda - 2k^2la_1\mu - k^2la_1\lambda^2 - 2lka_1a_0\lambda$$
$$-4k^2b_2\mu - 2k^2b_1\lambda - 2kla_1{}^2\mu = 0$$

$$(\frac{G'}{G})^2 : -2kla_1{}^2\lambda - 4k^2b_2\lambda - 2kla_1a_0 - lsa_1 - 2k^2b_1 - 3lk^2a_1\lambda = 0$$

$$(\frac{G'}{G})^3 : -2kla_1{}^2 - 4k^2b_2 - 2k^2la_1 = 0$$

For Eq.(3.7):

$$(\frac{G'}{G})^0 : sb_0 + 2kb_0a_0 - g_2 - k^2b_1\mu = 0$$

$$(\frac{G'}{G})^1 : -k^2b_1\lambda + sb_1 + 2kb_1a_0 - 2k^2b_2\mu + 2ka_1b_0 = 0$$

$$(\frac{G'}{G})^2 : -k^2b_1 - 2k^2b_2\lambda + 2kb_2a_0 + 2kb_1a_1 + sb_2 = 0$$

$$(\frac{G'}{G})^3 : -2k^2b_2 + 2ka_1b_2 = 0$$

Solving the algebraic equations above, yields:

$$a_1 = k, a_0 = a_0, b_2 = -kl, b_1 = -kl\lambda, b_0 = -kl\mu$$
$$k = k, l = l, s = k^2\lambda - 2ka_0, g_1 = g_2 = 0 \quad (3.13)$$

Where a_0, k, l are arbitrary constants.

Substituting (3.13) into (3.11) and (3.12), yields:

$$u(\xi) = k(\tfrac{G'}{G}) + a_0 \quad (3.14)$$

$$v(\xi) = -kl(\tfrac{G'}{G})^2 - kl\lambda(\tfrac{G'}{G})^1 - kl\mu \quad (3.15)$$

where $\xi = kx + ly + (k^2\lambda - 2ka_0)t$.

Substituting the general solutions of (3.10) into (3.14) and(3.15), we have:

When $\lambda^2 - 4\mu > 0$

$$u_1(\xi) = -\frac{k\lambda}{2} + \frac{k\sqrt{\lambda^2-4\mu}}{2}.$$
$$\left(\frac{C_1 \sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\right) + a_0$$

$$v_1(\xi)=\frac{kl\lambda^2}{4}-\frac{kl}{4}(\lambda^2-4\mu).$$

$$(\frac{C_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi})^2-kl\mu$$

where $\xi=kx+ly+(k^2\lambda-2ka_0)t$, a_0,k,l,C_1,C_2 are arbitrary constants.

When $\lambda^2-4\mu<0$

$$u_2(\xi)=-\frac{k\lambda}{2}+\frac{k\sqrt{4\mu-\lambda^2}}{2}.$$

$$(\frac{C_1\sinh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\cosh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\cosh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\sinh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi})+a_0$$

$$v_2(\xi)=\frac{kl\lambda^2}{4}-\frac{kl}{4}(\lambda^2-4\mu).$$

$$(\frac{C_1\sinh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\cosh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\cosh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\sinh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi})^2-kl\mu$$

where $\xi=kx+ly+(k^2\lambda-2ka_0)t$, a_0,k,l,C_1,C_2 are arbitrary constants.

When $\lambda^2-4\mu=0$

$$u_3(\xi)=\frac{k(2C_2-C_1\lambda-C_2\lambda\xi)}{2(C_1+C_2\xi)}+a_0$$

$$v_3(\xi)=\frac{k\lambda^2 l}{4}-\frac{klC_2^{\,2}}{(C_1+C_2\xi)^2}-kl\mu$$

where $\xi=kx-\frac{3b_1^{\,2}}{2k}t$, a_0,k,l,C_1,C_2 are arbitrary constants.

IV. CONCLUSION

The main points of the (G'/G)-expansion method are thatassuming the solution of the ODE reduced by using thetraveling wave variable as well as integrating can be expre-ssedby an m-th degree polynomial in (G'/G), where $G=G(\xi)$ is the general solutions of a second order LODE.The positive integer m is determined by the homogeneousbalance between the highest order derivatives and nonlinearterms appearing in the reduced ODE, and the coefficientsof the polynomial can be obtained by solving a set ofsimu-ltaneous algebraic equations resulted from the processof using the method. Furthermore the method can also beused to many other nonlinear equations.

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