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On $F_a(3,-1)$- Structure Manifold Defined By A Tensor Field $F(\neq 0)$ of Type (1, 1) Satisfying $F^3 - a^2F = 0$

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ABSTRACT
Structures defined by a tensor field f of type (1,1) satisfying $f^3 + f = 0$ have been studied by Prof. Yano[5] and others. Several structures defined by (1,1) tensor field ϕ satisfying $\phi^4 \pm \phi^2 = 0, \quad \phi^{2k} \pm \phi^{2k-2} = 0$ etc. have been studied by Yano, Gupta and others. In this paper, we have defined and studied $F_a(3,-1)$-structure manifolds. Some interesting results on a such a structure have been obtained.

I. INTRODUCTION: $F_a(3,-1)$-STRUCTURE

Let M^n be an n-dimensional differentiable manifold of differentiability class C^∞. Suppose there exists on M^n, a tensor field $F(\neq 0)$ of type (1,1) satisfying

$$F^3 - a^2F = 0 \tag{1.1}$$

Where 'a' is a complex number not equal to zero.
In M^n, let us put

$$l = \frac{F^2}{a^2} \text{ and } m = I_n - \frac{F^2}{a^2} \tag{1.2}$$

I_n denotes the unit tensor field. Then in view of the equation (2.1), and (2.2), it can be easily shown that

$$l^2 = l, \quad m^2 = m, \quad lm = ml = 0 \quad \text{and} \quad l + m = I_n \tag{1.3}$$

Thus for tensor field $F(\neq 0)$ of type (1,1) satisfying (2.1). The operators l and m defined by (2.2) when applied to the tangent space of M^n at a point are complementary projection operators. Thus there exist complementary distributions L^* and M^* corresponding to the projection operators l and m respectively. If the rank of F is constant everywhere and equal to r, the dimensions of L^* and M^* are r and $(n-r)$ respectively. Let us call such a structure on M^n as $F_a(3,-1)$- structure of rank r [4].

Theorem (1.1)
For (1,1) tensor field $F(\neq 0)$ satisfying (2.1) and for the operators l and m given by (2.2), we have

(i) $F \circ l = l \circ F = F$
(ii) $F \circ m = m \circ F = 0$
(1.4)
(iii) $F^2 \circ l = l \circ F^2 = a^2 l$
(iv) $F^2 \circ m = m \circ F^2 = 0$

Thus F acts on l as a GF-structure operator and on m as a null operator.

Proof
Proof follows easily by virtue of equations (2.1) and (2.2).

Theorem (1.2)
If $Rank(F) = n$, the manifold M^n admits a GF-structureconsequently.

$$l = I_n \text{ and } m = 0 \tag{1.5}$$

Proof
Since $Rank(F) = n$, F^{-1} exists. In the view of the equation (2.1), we can write

$$F(F^2 - a^2 I_n) = 0$$

Multiplying the above equation by F^{-1}, we get

$$F^2 = a^2 I_n \tag{2.6}$$

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Hence M^n admits a GF-structure. Again from the above equation (2.6)

$$l = \frac{F^2}{a^2} = I_n \text{ and}$$

$$m = I_n - \frac{F^2}{a^2} = 0$$

This proves the proposition.

Theorem (1.3)

If $Rank(F) = n-1$, the manifold M^n admits a general almost contact structureand operators l and m given by

$$l = I_n + \frac{l}{a^2} u \otimes U \text{ and } m = -\frac{l}{a^2} u \otimes U \quad \textbf{(1.7)}$$

where u is 1-Form and U a vector field on M^n.

Proof

In the view of the equation (2.1), we have

$$F(F^2 - a^2 I_n) = 0.$$

Since $Rank(F) = n-1$ there exists a vector field U and 1-Form u on M^n such that,

$F^2 - a^2 I_n = u \otimes U$ and $\overline{U} = 0$ where

$$\overline{U} = F(U). \quad \textbf{(1.8)}$$

Thus we have

(i) $F^2 = a^2 I_n + u \otimes U$ and **(1.9)**

(ii) $\overline{U} = 0$

Multiplying (2.9)-(i) by F and making use of (2.1), we get

$$u \circ F = 0 \quad \textbf{(1.10)}$$

Barring (2.9)-(ii) and making use of (2.9) itself, we get

$$u(U) = -a^2 \quad \textbf{(1.11)}$$

In the view of the equations (2.9), (2.10), and (2.11) it follows that the manifold M^n admits a general almost contact structure. Rest part of the proof is obvious.

Theorem (1.4)

In manifold M^n with $F_a(3,-1)$- structure, we have

$$\left(m + \frac{iF}{a}\right)\left(m - \frac{iF}{a}\right) = I_n \quad \textbf{(1.12)}$$

I_n denotes the unit tensor field.

Proof

Proof follows easily by the virtue of the equations (2.2) and (2.3).

Theorem (1.5)

In the $F_a(3,-1)$- structure of rank 2m, there are m eigen values each equal to a, m values each $-a$ *and* $(n-2m)$ eigen values each equal to zero of F.

Proof

Let λ be the eigen values of F and P the corresponding eigen vectors. So

$$F(P) = \lambda P, \quad F^2(P) = \lambda^2 P, \quad F^3(P) = \lambda^3 P, ...$$

Hence in view of the equation (2.1), we have

$$\lambda(\lambda^2 - a^2)P = 0,$$

which proves the proposition.

II. $F_a(3,-1)$ - METRIC STRUCTURE

We now assume that the manifold M^n is endowed with the Riemannian metric tensor 'g' satisfying

$$g(\overline{X}, Y) + g(X, \overline{Y}) = 0 \quad \textbf{(2.1)}$$

where $\overline{X} = F(X)$ and X, Y are arbitrary vector field on M^n. Let us call the $F_a(3,-1)$- structure admitting the Riemannian metric tensor 'g' satisfying (3.1) as $F_a(3,-1)$- metric structure.

Theorem (2.1)

In the manifold M^n admitting $F_a(3,-1)$- metric structure, the metric tensor 'g' satisfies

$$g(\overline{X}, \overline{Y}) + a^2 g(X, Y) = a^2 g(mX, Y) \quad \textbf{(2.2)}$$

Proof

Barring X in (3.1) and making use of (2.2), we get

$$g(a^2 X - a^2 mX, Y) + g(\overline{X}, \overline{Y}) = 0$$

or

$$a^2 g(X, Y) - a^2 g(mX, Y) + g(\overline{X}, \overline{Y}) = 0$$

or

$$g(\overline{X}, \overline{Y}) + a^2 g(X, Y) = a^2 g(mX, Y)$$

which proves the proposition.

Theorem (2.1)

$F_a(3,-1)$- metric structure is not unique. If we put

$$\mu F' \overset{def}{=} F\mu \text{ and } g'(X, Y) \overset{def}{=} g(\mu X, \mu Y) \quad \textbf{(2.3)}$$

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where μ is the non- singular (1,1) tensor field, then (F', g') also gives $F_a(3,-1)$- metric structure on M^n.

Proof

Post-multiplying (3.3)-(i) by F' and making use of the same equation, we obtain

$$\mu F'^2 = F^2 \mu$$

Thus in the view of above equation, we get

$$\mu F'^3 - a^2 \mu F' = F^3 \mu - a^2 F \mu \qquad \textbf{(2.4)}$$

In the view of equation (2.1) and (3.4), it follows that

$$F'^3 - a^2 F' = 0 \qquad \textbf{(2.5)}$$

Thus F' gives $F_a(3,-1)$- structure on M^n.

Again

$$g'(F'X, F'Y) = g(\mu F'X, \mu F'Y)$$

$$= g(F\mu X, F\mu Y)$$

By virtue of equation (3.2), the above equation takes the form

$$g'(F'X, F'Y) = -a^2 g(\mu X, \mu Y) + a^2 g(m\mu X, \mu Y)$$

(2.6)

Now

$$m\mu X = \mu X - \frac{\mu \overline{\overline{X}}}{a^2}$$

$$= \mu X - \mu \frac{F'^2 X}{a^2}$$

in the view of (3.3)(i)

Thus the equation (3.6) takes the form

$$g'(F'X, F'Y) + a^2 g'(X, Y) = a^2 g'(mX, Y)$$

(2.7)

Thus (F', g') gives the $F_a(3,-1)$- metric structure over the manifold M^n.

III. CONCLUSION

$F_a(3,-1)$- structure has been studied and on the basis of rank of F, its various forms are studied as GF- structure, general almost contact structure, and metric structure by using Riemannian metric tensor g.

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