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Modelling of Fractional Blood Flow as Non-Newtonian Fluid of Jeffry Type through a Stenosis Arteries

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ABSTRACT
Fractional electro-hydrodynamic blood flow with magnetic nanoparticles through the arteries with applied magnetic field as a non-Newtonian fluid of Jeffrey model was modeled, and the analytical solution of the fractional differential equation obtained from the modeling was obtain by the use of Laplace and Henkel transforms. The numerical solution is obtained to compare the effect of Womersley parameter, Hartmann number, and Jeffrey parameter, on the motion of the fluid and magnetic nanoparticles between the fractional fluid model and the classical fluid model.

Key words:Non-Newtonian Fluid of Jeffry Type, Fractional Model,Stenosis Arteries

I. INTRODUCTION

In the last few decades, researchers consider a lot of bio-fluids (blood) as a non-Newtonian fluid because of their rheology in nature, blood flow is essential in maintaining life due to its importance in transporting oxygen and nutrients to all parts of the body and removing metabolic waste away from the cells. Blood is considered as a non-Newtonian fluid due to it complex dynamic processes which contribute to the shear-thinning, viscoelastic and thixotropic behavior. The application of simulation of blood flow is very important in the process of decision making during treatment of cardiovascular diseases such as Atherosclerosis, Aneurysms, and Stenosis and so on, which are the major causes of mortality and morbidity in the developed world (Morovec and Liepsch 1983, Chaturani and Pannalagar 1985, Janel and Moura 2010, Chen and Lu 2004, Bernsdorf and Wang 2009, Mandal 2015 and Padma et al 2019). Blood composed of blood cells in blood plasma (Plasma which constitute 55% of blood fluid, is mostly water 92% by volume), and contains dissipated proteins, glucose, mineral ions hormones and the blood cell. The blood is a concentrated suspension of cellular element which are in aqueous solution with the following constituent, red blood cell which contains negative charge carriers that creates magnetic field on the wall of the artery, white blood cell, platelets, and the plasma contain electrolytes and organic molecules (Mandal 2015, Padma et al 2019, Yakubu et al 2020, and Yakubu et al 2021). To capture the rheological response of the blood over some physiological conditions, an accurate constitutive mathematical model is required in order to have meaningful hemodynamic simulations Janel and Moura (2010).In recent decade, researchers become more interested in fractional calculus, for its applications in various scientific, engineering system, biological science and medical diagnostics Abdulhameed et al (2017), Ionescu et al (2017) and Baleanu et al (2012).

Due to their widely application in different field, fractional derivatives has been documented in a lot of literatures for their flexibility in describing the behaviors of non-Newtonian fluid Ionescu et al (2017), Machado et al (2011), Liu et al (2011), Nadeem (2007), Haitao (2009), Nauman et al (2019) and Jinghua et al (2019). And the noteworthy model among the non-Newtonian fluids with convective derivative on the phenomena of relaxation and retardation time is the Jeffrey model.

Many studieswere conducted on Jeffrey model. See examples:Ponalagusamy(2017),Priyadharshini and Ponalagusamy (2017), Akbar et al (2011a), Nallapu and Radhakrishnamacharya(2014), Ponalagusamy (2016), Sharma and Yadava (2017), Hayat et al (2015), Akbar et al (2011b), Vajravelu et al (2011) and Akbar et al (2012).

Recently, the world of Mathematics is moving towards fractional calculus (fractional derivatives) because it application toward given a better and general description of the issue in questions.Based on the aforementioned rationales, the model presented by Padma et al (2019) was limited to the classical model of non-Newtonian fluid of Jeffrey type only. In order to improve the model, the present study aimed at extendingthe work of Padma et al (2019) toa fractional order derivative because of its advantage over a classical model.Besides, the above literature considered blood as a classical non-Newtonian fluid and Padma et al (2019) consider magnetic field, magnetic nanoparticles and applied electric field as classical fluid, while Yakubu et al (2020) and Yakubu et al (2021) consider blood as fractional non-Newtonianfluid (Maxwell and Burger's fluids). However, despite relevant studies on Jeffery fluid model, the effect of electric field on the model has not being into consideration, hence this arose the researchers' attention in coupling blood as a fractional Jeffery fluidand the effect of electric field in the present study.

II. MODELING OF THE PROBLEM

2.1 Schematic diagram of the problem

The studyconsiderered incompressible, pulsatile, laminar and axisymmetric blood flow in smaller arteries with low shear rate having a flow pattern of non-Newtonian fluid of Jeffrey model with magnetic nanoparticles through a cylindrical arteries segment of length L. The cylindrical coordinatesare considered to represent the direction of the flow in axial, radial and circumference directions respectively.

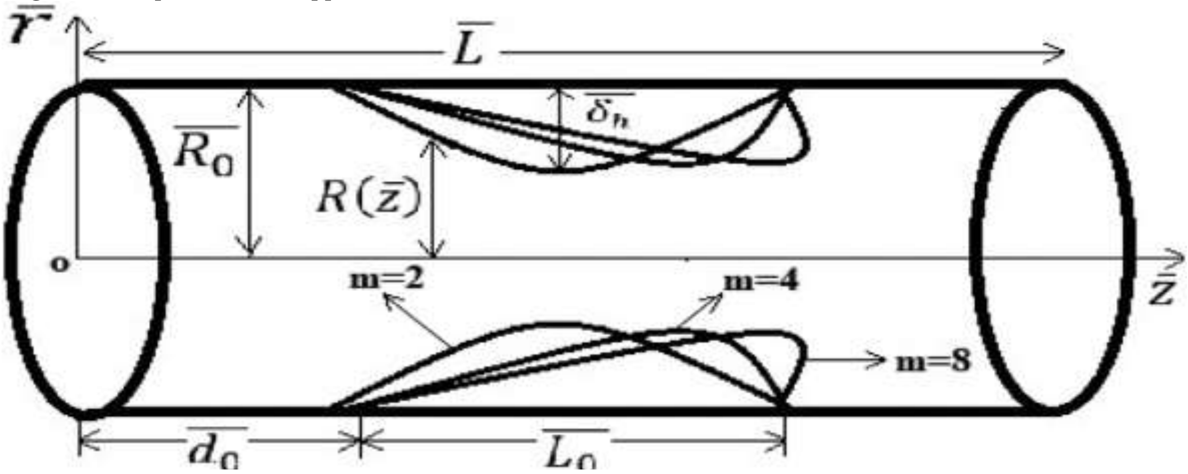

Figure 1: Geometry of the Non-Tapered Stenosed Artery with Different Shape Parameter m = 2,4 and 8

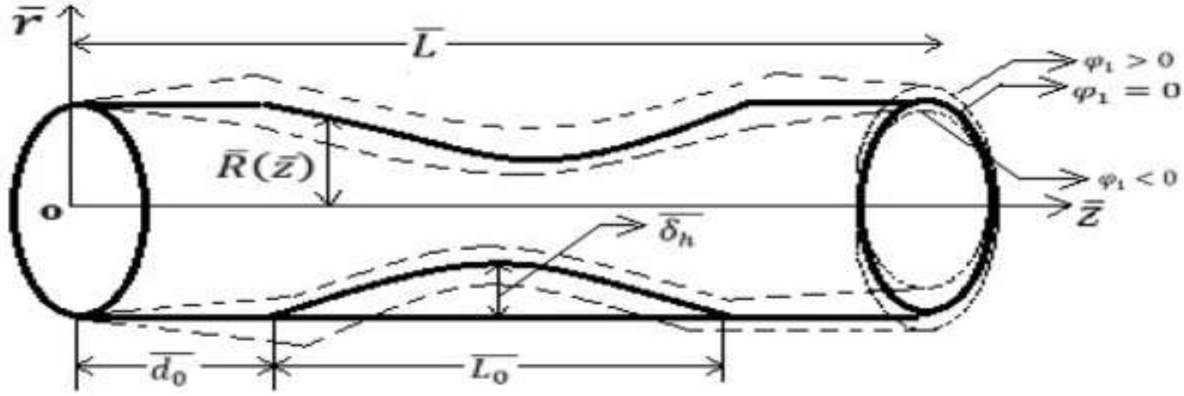

Figure 2: Geometry of the Tapered Stenosed Artery with Different Tapered Angle

2.2.1 Constitutive equation of the Jeffery fluid Model

The Naiver-Stoke equation governing the blood motion, the constitutive equation of the Jeffrey fluid model and the magnetic field equations are defining as:

$$\nabla \bullet \overline{B} = 0, \nabla \times \overline{B} = \overline{\mu_0}\overline{J}, \nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial \overline{t}} \qquad (1)$$

where $\overline{B}$ and $\overline{E}$ represent the flux intensity of the magnetic and electric fields respectively,
where $\overline{J}$ is the density of the current which can be obtained from the generalized Ohm's law as:

$$\overline{J} = \overline{\sigma}\left(\overline{E} + \overline{V} + \overline{B}\right), \qquad (2)$$

where $\overline{V}$ and $\overline{\sigma}$ are the flow velocity field and the electric conductivity respectively.
The resultant electromagnetic body force acting on the on the blood is given by:

$$\overline{F}_{emf} = \overline{\rho_e}\overline{E} + \overline{J} \times \overline{B} = \overline{\rho_e}\overline{E} + \overline{\sigma}\left(\overline{E} + \overline{V} + \overline{B}\right) \times \overline{B} \qquad (3)$$

where $\overline{u}_f, \overline{\rho}_e$ and $\overline{E}_{\bar{z}}$ are the fluid velocity in the axial direction, the free electric charge density and the component of external electric field in the axial direction respectively. Hence the net electromagnetic body force introduced on the momentum equation is given by:

$$\overline{F}_{emf} = \overline{\rho_e}\overline{E_{\bar{z}}} - \overline{\sigma}\overline{B_0}^2\overline{u}_f \qquad (4)$$

2.3 Basic Flow Equations

When fluid is in motion, it must move in such a way that mass is conserved. In fluid dynamics, the continuity equation states that the rate at which mass enters a system is equal to the rate at which mass leaves the system plus the accumulation of mass within the system. The differential form of the continuity and momentum equation of the unsteady fluid flow of Jeffrey fluid are described as:

$$\overline{\rho}\left[\frac{\partial}{\partial \bar{t}} + \overline{v}_f\frac{\partial}{\partial \bar{r}} + \overline{u}_f\frac{\partial}{\partial \bar{z}}\right]\overline{v}_f = -\frac{\partial \overline{P}}{\partial \bar{r}} + \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\overline{S}_{\bar{r}\bar{r}}f\right) + \frac{\partial}{\partial \bar{z}}\left(\overline{S}_{\bar{r}\bar{z}}f\right) - \frac{\overline{S}_{\theta\theta f}}{\bar{r}} \qquad (5)$$

$$\overline{\rho}\left[\frac{\partial}{\partial \bar{t}} + \overline{v}_f\frac{\partial}{\partial \bar{r}} + \overline{u}_f\frac{\partial}{\partial \bar{z}}\right]\overline{u}_f = -\frac{\partial \overline{P}}{\partial \bar{z}} + \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\overline{S}_{\bar{z}\bar{r}}f\right) + \frac{\partial}{\partial \bar{z}}\left(\overline{S}_{\bar{z}\bar{z}}f\right) + \overline{\rho}_e\overline{E}_{\bar{z}} \qquad (6)$$
$$-\overline{\sigma}\overline{B_0}^2\overline{u} + \overline{K}_s\overline{N}_p\left(\overline{u}_p - \overline{u}_f\right) + \overline{G}\left(\bar{t}\right)$$

$$\frac{\partial \overline{v}_f}{\partial \bar{r}} + \frac{\overline{v}_f}{\bar{r}} + \frac{\partial \overline{u}_f}{\partial \bar{z}} = 0 \qquad (7)$$

And the extra stress tensors of Jeffrey fluid are given by:

$$\overline{S}_{\bar{r}\bar{r}}f = \frac{2\overline{\mu}}{1+\lambda_1}\left[1 + \overline{\lambda}_2\left(\overline{v}_f\frac{\partial}{\partial \bar{r}} + \overline{u}_f\frac{\partial}{\partial \bar{z}}\right)\right]\frac{\partial \overline{v}_f}{\partial \bar{r}}$$

$$\overline{S}_{\bar{r}\bar{z}} = \overline{S}_{\bar{z}\bar{r}} = \frac{\overline{\mu}}{1+\lambda_1}\left[1 + \overline{\lambda}_2\left(\overline{v}_f\frac{\partial}{\partial \bar{r}} + \overline{u}_f\frac{\partial}{\partial \bar{z}}\right)\right]\left(\frac{\partial \overline{v}_f}{\partial \bar{z}} + \frac{\partial \overline{u}_f}{\partial \bar{r}}\right)$$

$$\overline{S}_{\bar{\theta}\bar{\vartheta}}f = \frac{2\overline{\mu}}{1+\lambda_1}\left[1 + \overline{\lambda}_2\left(\overline{v}_f\frac{\partial}{\partial \bar{r}} + \overline{u}_f\frac{\partial}{\partial \bar{z}}\right)\right]\frac{\overline{v}_f}{\bar{r}} \qquad (8)$$

$$\overline{S}_{\bar{z}\bar{z}}f = \frac{2\overline{\mu}}{1+\lambda_1}\left[1 + \overline{\lambda}_2\left(\overline{v}_f\frac{\partial}{\partial \bar{r}} + \overline{u}_f\frac{\partial}{\partial \bar{z}}\right)\right]\frac{\partial \overline{u}_f}{\partial \bar{z}}$$

where $\bar{u}_f$ and $\bar{v}_f$ denote the fluid velocity components in the axial and radial direction, $\bar{\rho}_e$ is the density of the fluid suspension, $\bar{P}$ is the pressure, λ_1 and $\bar{\lambda}_2$ are the retardation time and the ratio of relaxation and retardation time respectively, $\bar{B}_0$, the applied uniform magnetic field, $\bar{\mu}$ is the dynamic viscosity of the fluid, $\bar{K}_s$ is the Stokes constant, $\bar{N}_p$ is the number of magnetic nanoparticles per unit volume of blood and $\frac{\bar{K}_s\bar{N}_p\left(\bar{u}_p-\bar{u}_f\right)}{\rho}$ is the interaction force between motion of particles and fluid and $\bar{G}\left(\bar{t}\right)$ is the periodic body force.

Then the momentum equation governing the flow of fluid in the cylindrical coordinate system is given by [Ponalagusamy (2016) & Chakraborty (2006)]:

$$\bar{\rho}\frac{\partial\bar{u}_f}{\partial\bar{t}}=\frac{\partial\bar{P}}{\partial\bar{z}}+\frac{\bar{\mu}}{1+\lambda_1}\left[\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left(\bar{r}\frac{\partial\bar{u}}{\partial\bar{r}}\right)\right]+\bar{K}_s\bar{N}_p\left(\bar{u}_p-\bar{u}_f\right)+\bar{\rho}_e\bar{E}_{\bar{z}}-\bar{\sigma}\bar{B}_0^2\bar{u} \tag{9}$$

and a_0, a_1 are the amplitude of steady pulsatile component of pressure gradient respectively and $\bar{\omega}_p=2\pi\bar{f}_p.\bar{f}_p$ is the frequency of pulse.

Assuming that the particles are having effective hydrodynamic radius $\bar{E}_{hyd.p}$ are uniformly distributed in the fluid and flow freely along with the fluid. There are number of forces acting on the particle such as magnetic force, buoyancy force and fluidic force through which we obtain the notion of the nanoparticles momentum equation in the axial direction as:

$$\bar{M}_p\frac{\partial\bar{u}_p}{\partial\bar{z}}=\sum\bar{F}_{tex} \tag{10}$$

where $\sum\bar{F}_{tex}$ is the total force exerted on the particles and equal to the sum of magnetic, buoyancy and fluidic forces. Omitting the buoyancy force, buoyancy force, believing that the particle experience only the fluidic force, we get the expression for the flow governing equation of the particles with respect to Stokes's law as [mirza et al (2017)].

$$\bar{M}_p\frac{\partial\bar{u}_p}{\partial\bar{z}}=\bar{K}_s\left(\bar{u}_f-\bar{u}_p\right) \tag{11}$$

where $\bar{M}_p$ is the mass of single particle, $\bar{u}_p-\bar{u}_f$ is the relative velocity and $\bar{K}_s=6\pi\bar{\mu}\bar{E}_{hyd.p}\left(\bar{u}_f-\bar{u}_p\right)$ is the Stokes constant and the negative sign indicate that the motion of the fluid and particles are in opposite direction.

The corresponding initial and boundary conditions are for the proposed problem as:

$$\bar{u}\left(\bar{r},0\right)=0,\frac{\partial\bar{u}\left(\bar{r},0\right)}{\partial\bar{t}}=0,\bar{u}\left(1,\bar{t}\right)=0 \tag{12}$$

With the aid of electrostatics theory, the relationship between the net charge density $\bar{\rho}_e$ and the potential distribution $\bar{\psi}\left(\bar{r}\right)$ is given by the expression

$$\nabla^2\bar{\psi}\left(\bar{r}\right)=\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left(\bar{r}\frac{\partial\bar{\psi}\left(\bar{r}\right)}{\partial\bar{r}}\right)+\frac{1}{\bar{r}^2}\frac{\partial^2\bar{\psi}\left(\bar{r}\right)}{\partial\bar{z}}=\frac{\bar{\rho}_e\left(\bar{r}\right)}{\in} \tag{13}$$

Known as the Boltzmann equation with the boundary condition and the net charge density

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$$\bar{\psi}(1)=\bar{\psi}_w \text{ and } \frac{\partial\bar{\psi}}{\partial\bar{r}}=0, \text{ at } \bar{r}=0 \quad (14)$$

$$\bar{\rho}_e(\bar{r})=e_0\left(n^+-n^-\right)=\frac{-2z_0^2e_0^2n_0\bar{\psi}(\bar{r})}{k_gT_e} \quad (15)$$

where $\in$ is the dielectric constant, $\bar{\psi}_w$ is the potential on the arterial wall, $z_0, n_0, e_0, k_g, T_e, n^+$ and n^- are the ion valence, concentration of ions, the electronic charge, the Boltzmann constant, the local absolute temperature of the fluid, the density number of cations and anions respectively.

Using Debye-Huckel parameter $\bar{k}^2=\frac{2z_0^2e_0^2n_0\bar{\psi}(\bar{r})}{\in k_gT_e}$ and linearized the Boltzmann equation we get a potential equation as:

$$\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left(\bar{r}\frac{\partial\bar{\psi}(\bar{r})}{\partial\bar{r}}\right)=\bar{k}^2\bar{\psi}(\bar{r}) \quad (16)$$

The introducing dimensionless parameter to the flow governing equations are:

$$u_f=\frac{\bar{u}_f}{u_0}, u_p=\frac{\bar{u}_p}{u_0}, r=\frac{\bar{r}}{R_0}, z=\frac{\bar{z}}{R_0}, t=\bar{\omega}_p\bar{t}, P=\frac{\bar{P}R_0^2}{\mu u_0},$$
$$a_0=\frac{\bar{a}_0R_0}{\mu u_0}, a_1=\frac{\bar{a}_1R_0^2}{\mu u_0} S=\frac{\bar{S}R_0^2}{\mu u_0}, \psi=\frac{\bar{\psi}}{\bar{\psi}_w} \quad (17)$$

Applying equation (17) to the above flow equations subject to the dimensionless initial and boundary conditions and dropping the bars yield to the following equations:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi(r)}{\partial r}\right)-K_e^2\psi(r)=0 \quad (18)$$

$$\beta^2\frac{\partial u_f}{\partial t}=-\frac{\partial P}{\partial z}+\frac{1}{1+\lambda_1}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_f}{\partial r}\right)\right]+R_c\left(u_p-u_f\right)-M^2u_f-K^2\psi(r) \quad (19)$$

$$\beta^2G\frac{\partial u_p}{\partial t}=u_f-u_p \quad (20)$$

$$\psi(1)=1, \frac{\partial\psi(0)}{\partial r}=0, u(r,0)=0, \frac{\partial u(r,0)}{\partial t}=0, u(1,t)=0 \quad (21)$$

where $\beta^2=\frac{\bar{\omega}_p\bar{\rho}R_0^2}{\mu}, R_c=\frac{\bar{K}_s\bar{N}_pR_0^2}{\mu}, G=\frac{\bar{M}_p\mu}{\bar{\rho}R_0^2\bar{K}_s}, M^2=\frac{\bar{\sigma}\bar{B}_0^2R_0^2}{\mu}$ and $K_e^2=\bar{k}^2R_0^2$ are the pulsatile Reynolds number (womersley parameter), particle concentration parameter, Hartmann number (magnetic parameter) and electrokinetic width parameter.

2.4 Solution Techniques

The solution to equation (18) subject to equation (21) is

$$\psi(r)=\frac{I_0(K_er)}{I_0(K)} \quad (22)$$

where I_0 is the modified Bessel function of first kind and order zero.

Hence, we will consider the time fractional momentum equations, based on Caputo-Fabrizio fractional derivative.

$$ {}^{CF}D_t^{\alpha}u(r,t)=\frac{1}{1-\alpha}\int_0^t\frac{\partial u(r,\tau)}{\partial\tau}exo\left(-\frac{\alpha(t-\tau)}{1-\alpha}\right)d\tau, 0<\alpha<1 $$

$$ L\{{}^{CF}D_t^{\alpha}u(r,t)\}=\frac{su(r,s)-u(r,0)}{(1-\alpha)s+\alpha} \quad (23) $$

Now the fractional differential equation with Caputo-Fabrizio derivative corresponding to momentum equation (18-20).

$$ \beta^2\,{}^{CF}D_t^{\alpha}u_f(r,t)=f(t)+\frac{1}{1+\lambda_1}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_f(r,t)}{\partial r}\right)\right]+R_c\left(u_p(r,t)-u_f(r,t)\right) \quad (24) $$

$$ -M^2u_f(r,t)-K^2\psi(r) $$

$$ \beta^2G\,{}^{CF}D_t^{\alpha}u_p(r,t)=u_f(r,t)-u_p(r,t) \quad (25) $$

where $f(t)=-\frac{\partial\overline{P}}{\partial\overline{z}}=a_0+a_1\cos(\omega t)$ (26)

Applying Laplace transform on equations (25) and (26), and using the definition in equation (23) we have:

$$ \beta^2\frac{su_f(r,s)-u(r,0)}{(1-\alpha)s+\alpha}=F(s)+\frac{1}{1+\lambda_1}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_f(r,s)}{\partial r}\right)\right]+R_c\left(u_p(r,s)-u_f(r,s)\right) $$

$$ -M^2u_f(r,s)-\frac{K^2}{s}\psi(r) $$

(27)

$$ \beta^2\frac{su_p(r,s)-u(r,0)}{(1-\alpha)s+\alpha}=u_f(r,s)-u_p(r,s) \quad (28) $$

where $u_f(r,s)=\int_0^{\infty}u_f(r,t)e^{-st}dt, u_p(r,s)=\int_0^{\infty}u_p(r,t)e^{-st}dt$ and $F(s)=\int_0^{\infty}f(t)e^{-st}dt$

$K^2\psi(r)$ is a constant with respect to time t.

Rearranging equations (27) and (28)

$$ \beta^2\frac{su_f(r,s)}{(1-\alpha)s+\alpha}=F(s)+\frac{1}{1+\lambda_1}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_f(r,s)}{\partial r}\right)\right]+R_c\left(u_p(r,s)-u_f(r,s)\right) \quad (29) $$

$$ -M^2u_f(r,s)-\frac{K^2}{s}\psi(r) $$

$$ \beta^2G\frac{su_p(r,s)}{(1-\alpha)s+\alpha}=u_p(r,s)-u_f(r,s) \quad (30) $$

Making $u_p(r,s)$ subject in equation (30) and substitute in equation (31), and rearranging we have

$$ u_p(r,s)=\left[\frac{(1-\alpha)s+\alpha}{\beta^2Gs+(1-\alpha)s+\alpha}\right]u_f(r,s) \quad (31) $$

$$ \left[\frac{\beta^2s+R_c\beta^2Gs+M^2[(1-\alpha)s+\alpha]}{(1-\alpha)s+\alpha}\right]u_f(r,s)=F(s)+\frac{1}{1+\lambda_1}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_f(r,s)}{\partial r}\right)\right]-\frac{K^2}{s}\psi(r) $$

(32)

Applying the finite Henkel transform of order zero to equation (34), we obtained

$$\left[\frac{\beta^2 s+R_c\beta^2 Gs+M^2[(1-\alpha)s+\alpha]}{(1-\alpha)s+\alpha}\right]u_f(\gamma_n,s)=F(s)\frac{J_1(\gamma_n)}{\gamma_n}$$
$$+\frac{1}{1+\lambda_1}\left[\gamma_n^2 u_f(\gamma_n,s)\right]-\frac{K^2}{s}\frac{\gamma_n}{\gamma_n^2+K^2}J_1(\gamma_n) \tag{33}$$

$$u_f(\gamma_n,s)=\left[\frac{[(1-\alpha)s+\alpha](1+\lambda_1)}{Q(s)}\right]\left[F(s)\frac{J_1(\gamma_n)}{\gamma_n}-\frac{K^2}{s}\frac{\gamma_n}{\gamma_n^2+K^2}J_1(\gamma_n)\right] \tag{34}$$

$$Q(s)=a_n s+b_n\alpha$$
$$a_n=\beta^2(1+\lambda_1)(1+R_cG)+M^2(1+\lambda_1)(1-\alpha)+\gamma_n^2(1-\alpha) \tag{35}$$
$$b_n=M^2(1+\lambda_1)+\gamma_n^2$$

Now applying the inverse Henkel transform on equation (34) we have

$$u_f(r,s)=\left[\frac{[(1-\alpha)s+\alpha](1+\lambda_1)}{Q(s)}\right]\left[F(s)\Omega(r)-\frac{1}{s}\Phi(r)\right] \tag{36}$$

where $\Omega(r)=2\sum_{n=1}^{\infty}\frac{J_0(\gamma_n r)}{\gamma_n J_1(\gamma_n)}$ and $\Phi(r)=2\sum_{n=1}^{\infty}\frac{K^2\gamma_n}{\gamma_n^2+K^2}\frac{J_0(\gamma_n r)}{J_1(\gamma_n)}$ respectively.

And for the velocity of the magnetic nanoparticles, we substitute equation (36) into equation (31) and obtained

$$u_p(r,s)=\left[\frac{(1-\alpha)s+\alpha}{\beta^2 Gs+(1-\alpha)s+\alpha}\right]\left[\frac{[(1-\alpha)s+\alpha](1+\lambda_1)}{Q(s)}\right]\left[F(s)\Omega(r)-\frac{1}{s}\Phi(r)\right] \tag{37}$$

The inverse Laplace transform of equation (36)-(37), with the aid of Gerby- Stephan's Algorithm were taken and the results are simulated graphically with the aid of MATCARD software.

III. RESULTS AND DISCUSSION

This section presented the numerical results simulated in graphical form for the fractional blood flow as a non-Newtonian fluid of Jeffery type through a stenosis artery.

3.1Velocity Field Profile

The blood flows with magnetic particle through an artery as a Jeffrey fluid were analyzed using Caputo-Fabrizio time fractional derivative under the influence of electric and magnetic fields. In order to have are good understanding of the effect of the fractional parameter on physiology of the flow pattern, the numerical results are presented graphically using Mathcad software for an explicit and noteworthy discussion. In this regard, we assume the following dimensionless physical parameters such as$a_0=0.5, a_1=0.5, \omega_p=\frac{\pi}{4}$.

An improved Mathematical modeling of fractional blood flow as a non-Newtonian blood fluid of Jeffery type through Stenosed arteries concerning the influence of externally imposed magnetic and electric filed are numerically simulated with help of graphs in Figure 3-10. For all computational simulations and their corresponding figures, the values of governing parameters are listed in section in all corresponding figures in question.However, we were interested, to analyze the influence of Fractional and Classical fluid parameter on the fluid flow velocity in all cases.

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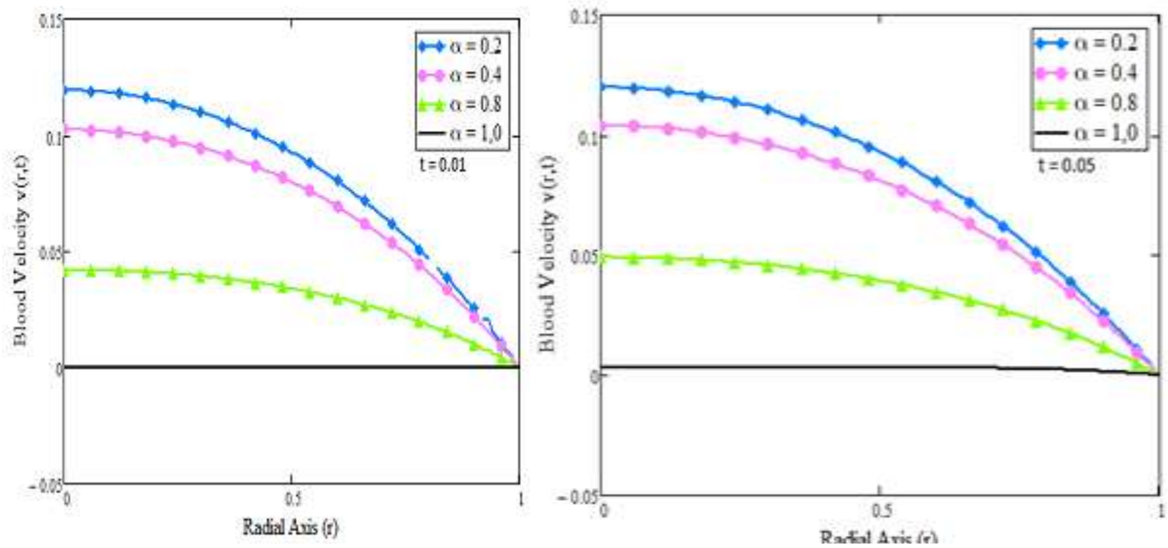
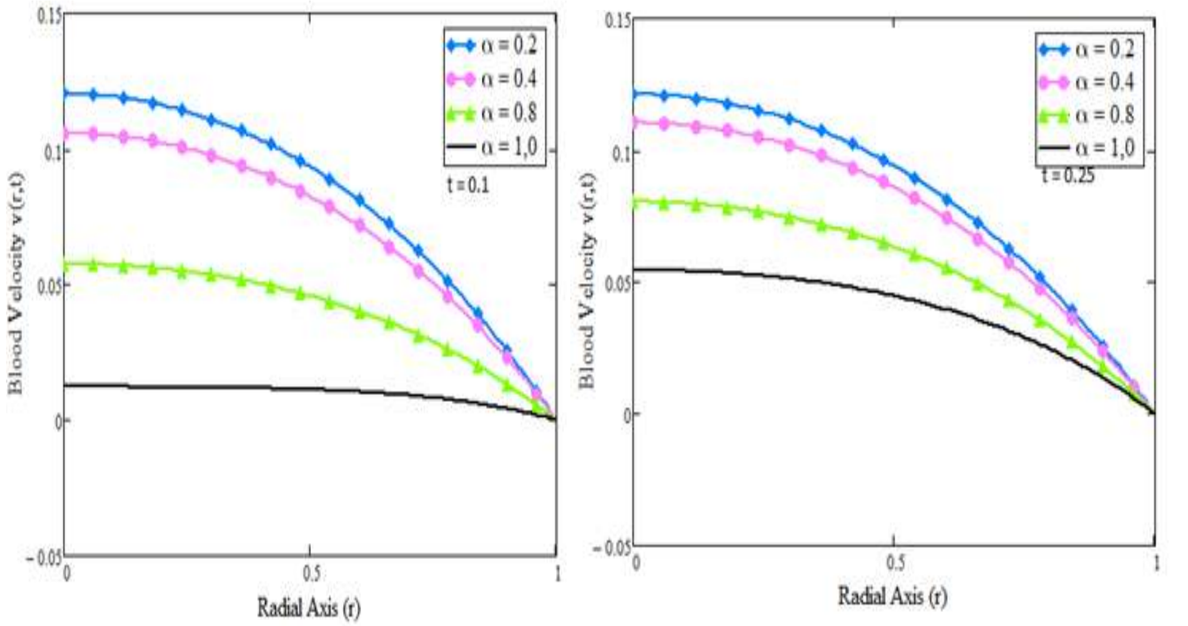


(a) t = 0.01 (b) t = 0.05



(c) t = 0.1 (d) t = 0.25

Figure 3: Dimensionless Fluid velocity distribution with axial distance for different fractional parameter for time t increment with $G = 0.8, \lambda_1 = 0.4, M = 1.0, K = 0.5, K = 0.5, R = 0.5, \beta = 0.7$.

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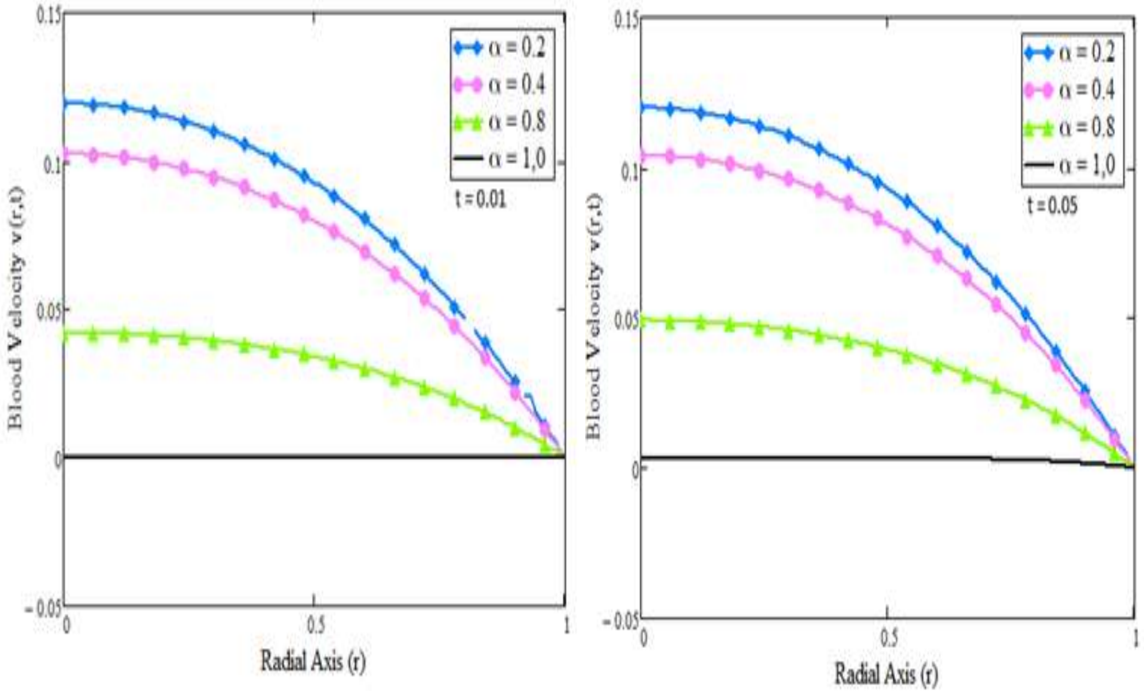
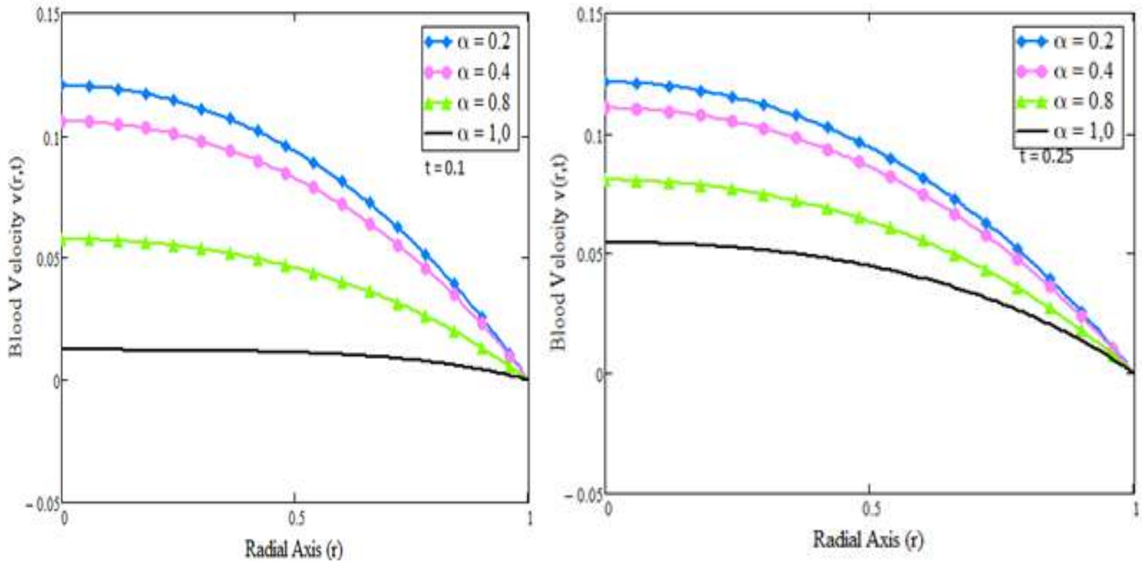


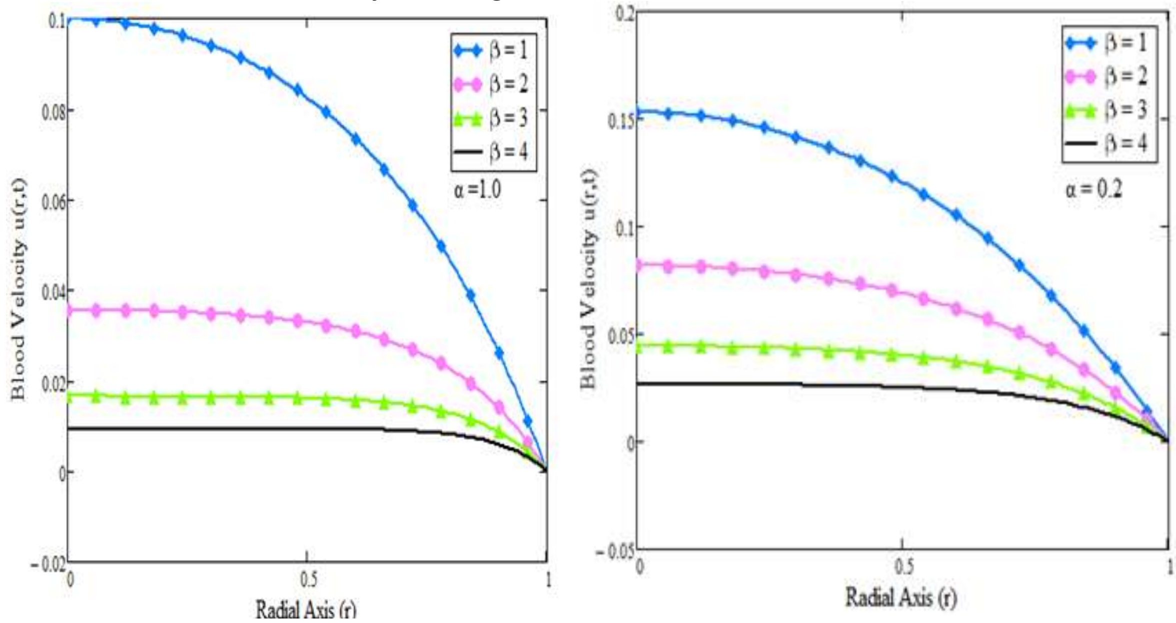
(a) t = 0.01 (b) t = 0.05



(c) t =0.1 (d) t = 0.25

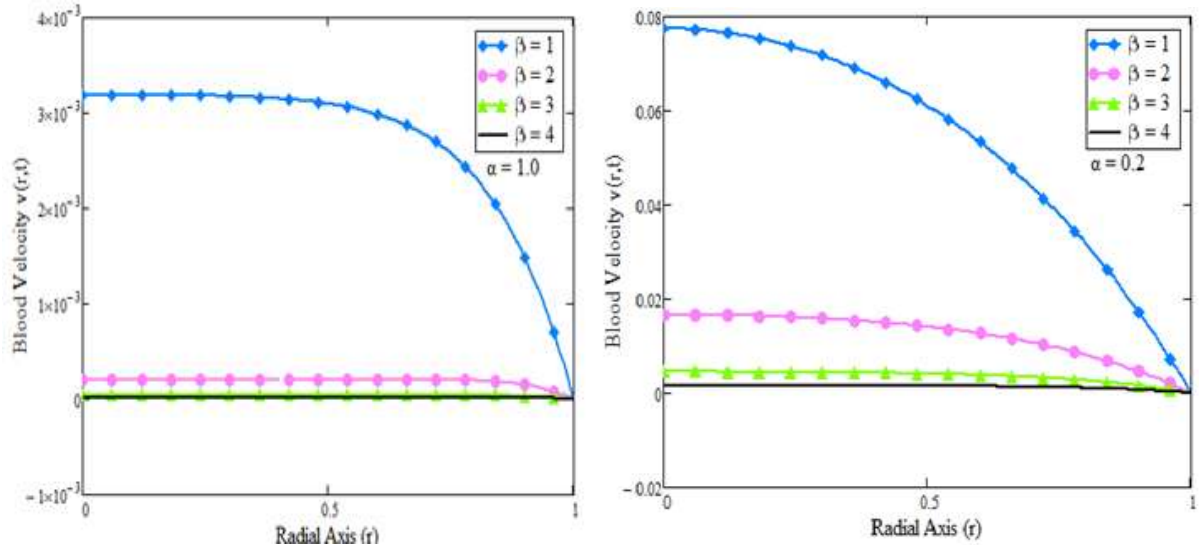
Figure 4: Dimensionless Particle velocity distribution with axial distance for different fractional parameter for time t increment with $G = 0.8, \lambda_1 = 0.4, M = 1.0, K = 0.5, K = 0.5, R = 0.5, \beta = 0.7$

Figure 3 & 4 shows the detail influence of the fractional parameter on the fluid and particle dimensionless velocity distribution against the axial distance, which depict that at smaller value of time t the fractional fluid move faster than the classical fluid while at larger values of time t the classical fluid is faster than the fractional fluid which is in agreement with Abdulhameed et al (2017) on fractional bio-fluid and Yakubu et al (2020) on fractional blood flow in an artery with magnetic effect.



(a) $\alpha = 1.0$ (b) $\alpha = 0.2$

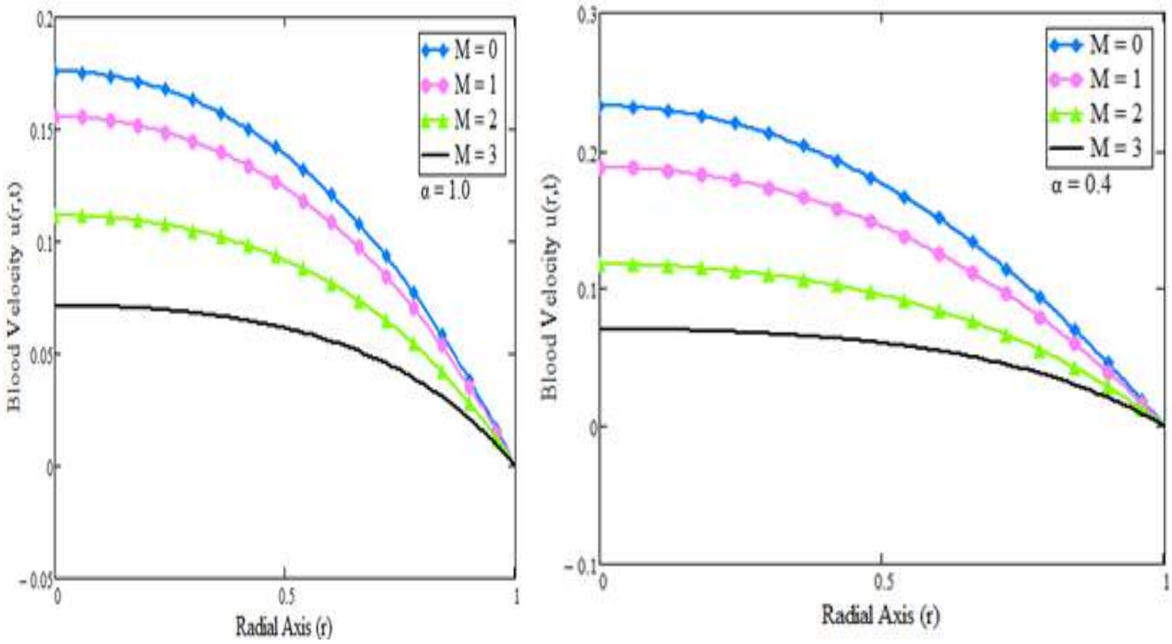
Figure 5: Dimensionless Fluid velocity distribution with axial distance for different pulsatile Reynolds number (Womersly parameter) for classical and fractional fluids with $\mathrm{G} = 0.8, \lambda_1 = 0.4, \mathrm{M} = 1.0, \mathrm{K} = 0.5, \mathrm{K} = 0.5, \mathrm{R} = 0.5$



(a) $\alpha = 1.0$ (b) $\alpha = 1.0$

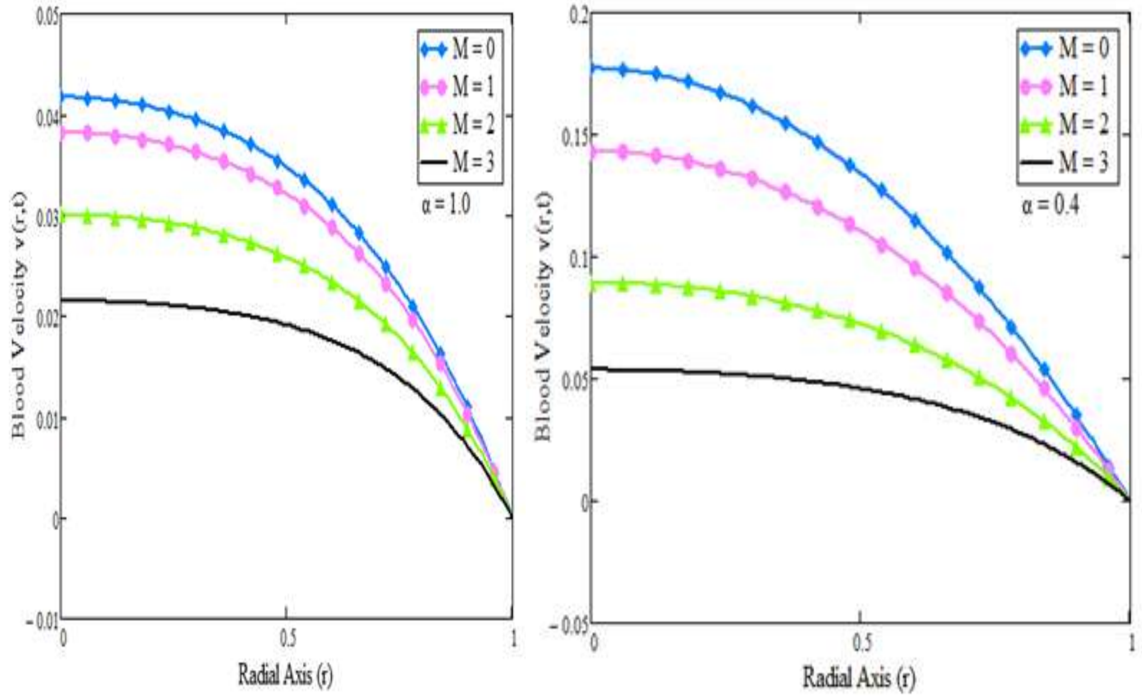
Figure 6: Dimensionless Particle velocity distribution with axial distance for different pulsatile Reynolds number (Womersly parameter) forclassical and fractional fluids with $\mathrm{G} = 0.8, \lambda_1 = 0.4, \mathrm{M} = 1.0, \mathrm{K} = 0.5, \mathrm{K} = 0.5, \mathrm{R} = 0.5$.

Figure 5 & 6 presented the effect of pulsatile Reynolds number (Womersly parameter) on the fluid and particle dimensionless velocity distribution against the axial distance, which indicated that the Womersly parameter has more effect on the velocity of the classical fluid than on the fractional fluid and the effect is more on the velocity of the particle then the velocity of the fluid.



(a) $\alpha = 1.0$ (b) $\alpha = 0.4$

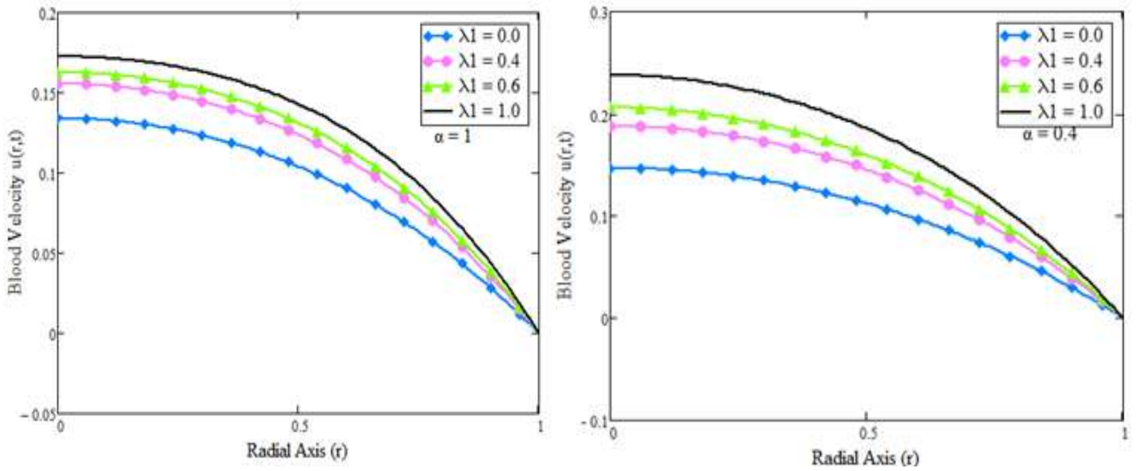
Figure 7: Dimensionless Fluid velocity distribution with axial distance for different Hartmann number with $G = 0.8, \lambda_1 = 0.4, K = 0.5, K = 0.5, R = 0.5, \beta = 0.7$.



(a) $\alpha = 1$ (b) $\alpha = 0.4$

Figure 8: Dimensionless Particles velocity distribution with axial distance for different Hartmann number with $G = 0.8, \lambda_1 = 0.4, K = 0.5, K = 0.5, R = 0.5, \beta = 0.7$

The effect of the magnetic field on the dimensionless velocity of the fluid and particle is shown on figure 7 and 8, where the result revealed that magnitude of the dimensionless velocity of the fractional fluid is more than that of the classical fluid due to the magnetic field effect which decelerates the velocity due to the induce Lorentz force that opposed the motion of the fluid and particle, and it shows that that the induced Lorentz force has greater effect on the particle because of their magnetic nature.



(a) $\alpha = 1$ (b) $\alpha = 0.4$

Figure 9: Dimensionless Fluid velocity distribution with axial distance for different Jeffrey parameter with $G = 0.8, M = 1.0, K = 0.5, K = 0.5, R = 0.5, \beta = 0.7$.

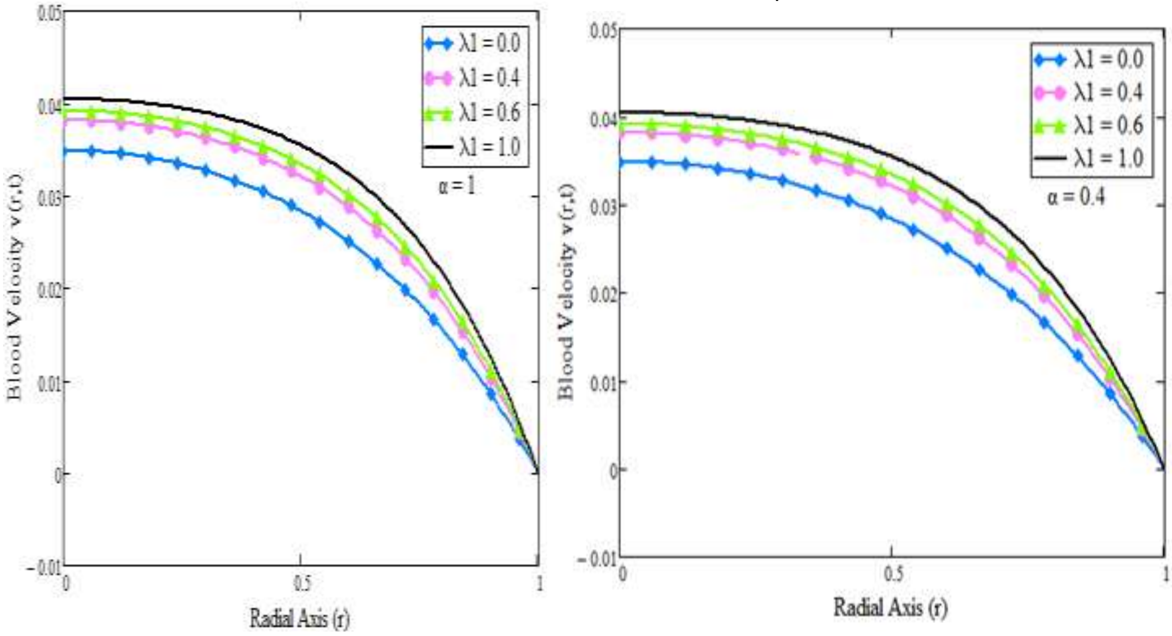


Figure 10: Dimensionless Particle velocity distribution with axial distance for different Jeffrey parameter with $G = 0.8, M = 1.0, K = 0.5, K = 0.5, R = 0.5, \beta = 0.7$.

Figure 9 & 10 are plotted to show the effect of the Jeffrey parameter on the dimensionless velocity of the fluid and particles, where the graph indicated a slightest difference in the effect on the velocity of the fluid, which shows that the velocity of the fractional fluid is little higher than that of the classical fluid, but show no difference in velocity of the particle between the fractional fluid and the classical fluid.

IV. CONCLUSION

The present research work considered the Caputo-Fabrizio time fractional order derivative effect of the physiology electro hydrodynamic blood and magnetic nanoparticles flow as a non-Newtonian fluid of Jeffrey model in the arteries with applied magnetic field, where the analytic solution for the velocity profile were obtained for both the fluid and the nanoparticles. And these are some of our main findings as follows:

- For smaller time the influence of the fractional parameter is more significant than for higher

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time for both the fluid and the magnetic nanoparticles.

- The velocity of the fluid or the nanoparticle can be controlled by regulating the fractional parameter.
- Numerical results were obtained in graph to compare physiological behaviors of the fluid and magnetic nanoparticles for the fractional order model fluid with the classical model fluid.
- It was observed that for the Womersley parameters and Hartmann number the velocity of both fluid and particles, the fractional model fluid is faster than that of the classical model fluid, and for the Jeffrey parameter, it has the same effect on fluid but no difference on the nanoparticles models.

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