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Introduction to Functions and Some Basic Techniques of Sketching the Graph and their Real Life Applications

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ABSTRACT: This article is mainly concern on the different types of functions mostly used in calculus at school and college level. It focuses on the concept, simple definition and examples of the functions and some basic terms that can be used while sketching the graph of the function. This article consists of the systematic procedures for sketching the curve of Cartesian functions, polar functions and the parametric functions with some illustrations and some real life applications of the given function.

Key Words: Cartesian curves, Curve sketching, Parametric curves, Polar curves

I. INTRODUCTION

Concept of Function: The concept of function is one of the fundamental concepts of modern mathematics. The word or the term 'function' was literally introduced by Leibniz in 1673 to mean any quantity varying from point to point of a curve, like the length of the tangent or the normal. In 1714, he again used the word "function" to mean quantities that depend on a variable. In 1718, Johann Bernoulli used the function of expression made up of constants and a variable. At the last of eighteenth century, Euler came to regard a function as any equation made up of constants and variables. Euler introduces and made extensive use of the extremely important notation y = f (x) in 1734 (Kline, M 1972). The form of the definition of function that had been used until well into the twentieth century was formulated by Dirichlet (1805–1859). He stated that, if two variables x and y are so related that for each value of x there corresponds exactly one value of y, then y is said to be a (single-valued) function of x. He called x, the variable to which values are assigned at will, the independent variable, and y, the variable whose values depend on the values assigned to x, the dependent variable. He called the values assumed by x the domain of the function, and the corresponding values assumed by y the range of the function.

A function is an action performed by a person, device, or sector or to produces a result. Function remains more or less fixed whereas the purpose (which indicates intention or objective) generally changes. For example, the function of a hammer is to strike something nearby whereas its purpose may be anything like what to strike and why. A mathematical relationship in which a quantity (dependent variable) depends on or is determined by another quantity (independent variable) or quantities. The dependent variable is said to be a function of the independent variable(s).If something is done, or something happens, to the independent variable(s), it is reflected in the dependent variable. For example, expenditure is a function of income and for a wage earner; income is a function of two variables; per hour wage rate and number of hours worked. So it is a process or operation that is performed routinely to carry out a part of the mission of an organization.

In mathematics, a function is a relation in which every element in the domain corresponds to one and only one element in the range where a relation is a correspondence that matches up two sets of objects. The first set is called the domain and the set of all corresponding elements is called the range (Raymond et. al, 2009). A relation or an expression involving one or more variables is a function. So, everybody get functions all around us. For example, a functional relationship between quantities and costs of any goods always exists. We are executing functions at home, office and public places in a different ways. Functions are important as well necessary for interpretation the local and world demographics and population growth, which

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are critical to manage well and for planning and development.

Definition of Function: Function is an expression, rule, or law that defines a relationship between one variable (independent variable) and another variable (dependent variable). For example, the position of a planet is a function of time. This relationship is commonly symbolized as y = f(x). So, it is a relationship between two or more quantities, which is the basic quantitative tool for describing the relationships. A function is a rule that relates how one quantity depends on other quantities. Mathematically, a function of a variable x is a rule f that assigns to each value of x a unique number f(x) is called the value of the function at x. i. e. f(x) = 3x + 1; or y = f(x).We read as f of x. Here variable y is said to be a function of another variable x because they are related such that by giving a value to x we get one and only one value of y. Since the function y depends on the value of x then, variable y is known as dependent variable and x is known as independent variable. The modern definition of function was first given by the German mathematician Peter Dirichlet in 1837(Elstrodt, J. 2007). According to him "If a variable y is so related to a variable x that whenever a numerical value is assigned to x, there is a rule according to which a unique value of y is determined, then y is said to be a function of the independent variable x".

In the 'real-world,' functions are mathematical representations of many input-output situations. A function is simply a "machine" that generates some output in relation to the given input.

To understand the function, imagine a "function machine" which is so constructed that if you input x into the machine it will be automatically doubles and add 3. So that the result or output of the machine will be (function) f(x) = 2x+ 3. Sometimes, in place of f(x) = 2x+ 3, we write y = 2x+ 3, where it is understood that the value of y, the dependent variable, depends on our choice of x, the independent variable (Figure 1).

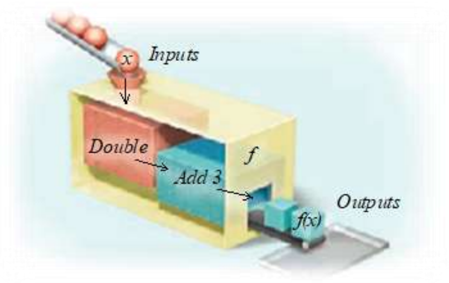

Figure 1

Graph of the Function: The graph of the function can be described in terms of ordered pairs. The order pairs of the function are simply the points on a graph, which are also described with an ordered pair of numbers. These ordered pair of the numbers is the major connection between graphs and functions. Every function that has a real-number domain and range has a graph, which is simply a pictorial representation of the ordered pairs of real numbers that make up the function. The sketch or graph of the functions is the visualized or displayed form of the function. A sketch depicts the important parts of a graph, it does not have to be to scale although it still has to be labeled correctly and any lines or points need to be correctly positioned in relation to each other and the axes. When functions are graphed, domain values are usually associated with the horizontal axis and range values with the vertical axis. In this regard, the graph of a function f is the graph of the equation y = f(x) where x is the independent variable and the first coordinate or abscissa of a point on the graph of f (Raymond, A. B., Michael, R. Z., & Karl, E. B., 2008). The variables y and f (x) can both be used to represent the dependent variable, and either one is the second coordinate or ordinate of a point on the graph of f (Figure 2).

The graph of the function is the set of all points (x, y) in the plane that satisfies the equation y = f(x). If the function is defined for only a few input values, then the graph of the function is only a few points, where the x-coordinate of each point is an input value and the y-coordinate of each point is the corresponding output value. It provides a means of displaying, interpreting and analyzing data in a visual form. Graphs can be used to model or described mathematically many real world situations using that helps everybody see the mathematics through its focus on visualization.

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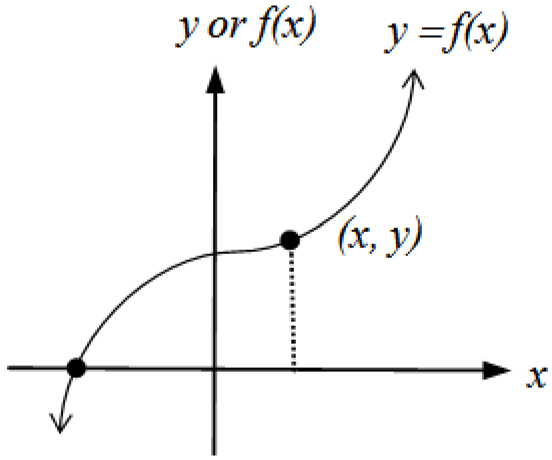

Figure 2

Objectives

This article aims to reflect the concept, meaning and definition of function, types of functions mostly used in calculus and some basic terms used while sketching the graph of the function. This also consists of the systematic procedures for sketching the different curves of common functions and their some real life applications.

Methodology

This article is mainly descriptive in nature followed by qualitative design. In this paper, the researcher has been adopted desk study method to search the secondary sources. This paper is based on the desk review of the published and unpublished literatures from different sources.

II. FINDINGS AND DISCUSSION

Types of Function

We have so many functions in our general practices. Here it is discuss the various types of function which are commonly used in calculus. They are as follows:

1. Constant function
2. Linear function
3. Quadratic function
4. Polynomial function
5. Rational function
6. Absolute value function
7. The square root function
8. Cube root function
9. The reciprocal function
10. Trigonometric function
11. Logarithmic function (Inverse of exponential function)
12. Parametric function

1. Constant Function

The function y = f(x) is said to be constant if y = c where c is constant. It is a special case of linear function where the value of m is zero or the highest power of x is zero. i. e. $y = 0.x + c$, or $y = m.x^0 + c \Rightarrow y = m + c$ where m and c both are constants. The graph of the constant function is always a straight line either parallel to x-axis or y-axis. In Figure 3, the constant function is parallel to x- axis.

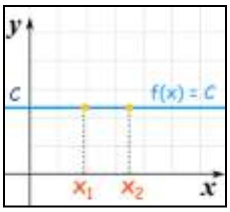

Figure 3(a)

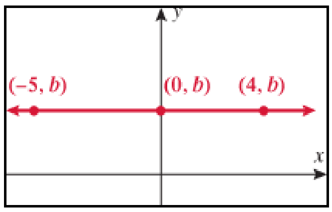

Figure 3(b)

2. Linear Function

The function y = f(x) is said to be linear function if it can be expressed in the form of y = m + c where m and c are constant i.e. y = 2x + 5.The graph of the linear function is always a straight line with slope m at x -axis and intercept c in y -axis from the origin. To find the intercept, we can put x = 0 for y- intercept and y = 0 for x- intercept. Then plotting the points on the axes we can sketch the graph.

Example 1: Identify the slope and the y-intercept of each line:

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i) $y = (2/3)\,x - 1$ ii) $y = 5$ iii) $x = -2$ iv) $2x - 3y = 9$

Solution:

i) $y = (2/3)\,x - 1$
This line has slope (2/3) and the y-intercept is the point (0, -1).

ii) $y = 5$
This is a horizontal line. The slope is 0, and the y-intercept is (0,5).

iii) $x = -2$
This is a vertical line. The slope is undefined, and the line does not cross the y-axis. (Note that this line is not a function)

iv) $y = 2/3x + 3$
The slope of this line is 2/3, and the y-intercept is the point (0, 3).

The graph below in Figure 4 shows these four lines together.

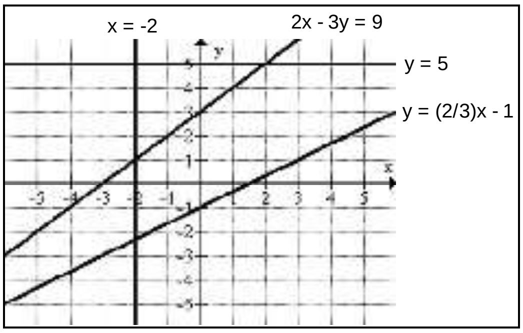


Figure 4

3. Quadratic Function

Any function of the form $y = ax^2 + bx + c$; where a, b and c are constants and $a \neq 0$ is called quadratic function. This function is also called square function whose equation contains the second power of x, x^2. For example, $f(x) = x^2 + 2x - 4$ is the square function. The graph of every square function is a parabola. A parabola has a vertex, and an axis of symmetry. The graph below in Figure 5 shows these aspects of the graph of $y = x^2$

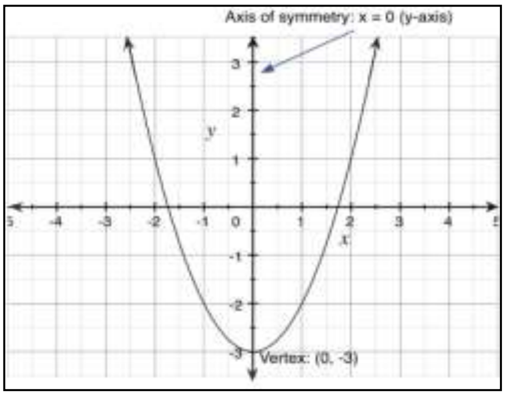


Figure 5

The techniques for sketching the curve of this function will be discussed later.

4. Polynomial Function

Any function of the form, $y = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots + a_0 x^0$ where n is non negative integer and $a_1 \ldots\ldots a_n$ are given constant numbers, is called polynomial function of degree n. i.e. i.e. $y = 4x^3 + 2x^2 - 3x + 4$ is the polynomial function of degree 3. Linear, quadratic and cubic functions are the special cases of polynomial function with degree 1, 2 and 3 respectively. The Figure 6 and Figure 7 are the polynomial function of degree 4 and 5 respectively.

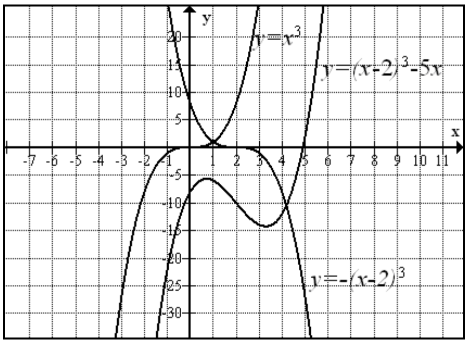


Figure 6

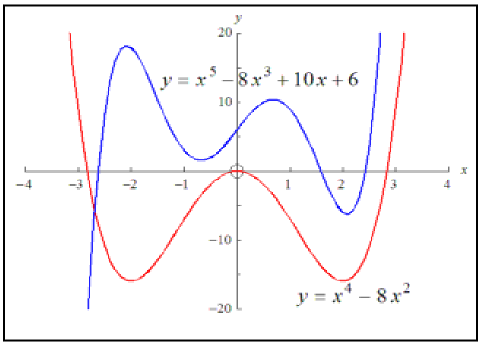
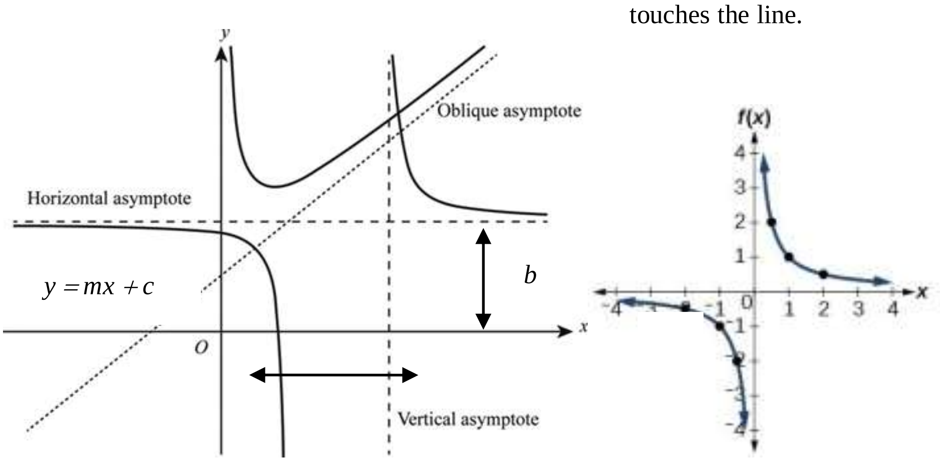


Figure 7

5. **Rational function:** The rational function is a function which can be expressed as the quotient of two polynomials. That is f(x) = $\frac{p(x)}{q(x)}$ where p(x) and q(x) are polynomials and where q(x) is not the zero polynomial. The domain of f consists of all inputs x for which q(x) ≠ 0. $y = \frac{x^2+1}{2x}$ is the rational function.

Rational functions contain three types of asymptotes horizontal, vertical and oblique, as given in the Figure 8 below. An asymptote is a line that the graph of a function approaches but never touches the line.



x	f(x)
−2	−0.5
−1	−1
−0.5	−2
0.5	2
1	1
2	0.5

Table 1

Figure 8

When x goes to ± ∞, the curve approaches some constant value b is the horizontal asymptote and when x approaches some constant value c from the left or right then the curve goes towards ± ∞ is the vertical asymptote. Likewise, when x goes to ± ∞, then the curve goes towards a line y = mx + c is the oblique asymptote where m is not zero as that is a horizontal asymptote.

6. **Absolute Value Function**

The function defined for all numbers x by y = |x| is called the absolute value function. The graph of the function is considered with the graph of the equation y = x for x ≥ 0. Let's first consider the parent of the family, y = |x|. Because the absolute value of a number is that number's distance from zero, all of the function values will be non-negative. If x = 0, then y = |0| = 0. If x is positive, then the function value is equal to x. For example, the graph contains the points (1, 1), (2, 2), and (3, 3), etc. However, when x is negative, the function value will be the opposite of the number. For example, the graph contains the points (-1, 1), (-2, 2), and (-3, 3), etc. As you can see in the graph in Figure 9, the absolute value function forms a "V" shape.

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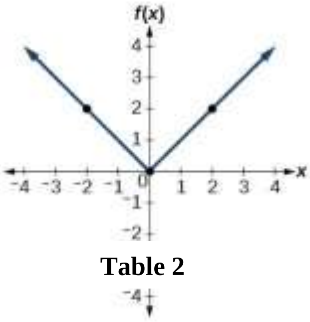


Table 2

x	f(x)
−2	2
0	0
2	2

Figure 9

7. The Square Root Function

The square root function is slightly more complicated. Consider for example, the parent of the family, $y = \sqrt{x}$. The domain of the function is limited to real numbers ≥ 0, as the square root of a negative number is not a real number. Similarly, the range of the function is limited to real numbers ≥ 0. This may seem confusing if you think of squares having two roots. For example, 9 have two roots: 3, and -3. However, for $y = \sqrt{x}$, we have to define the function value as the principal root, which means the positive root.

The graph of the function $y = \sqrt{x}$ is shown in Figure 10.

f(x) = √x	
x	f(x)
–2	undefined
–1	undefined
0	0
1	1
2	1.4142136
3	1.7320508
4	2

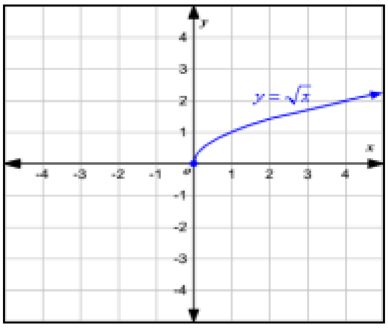
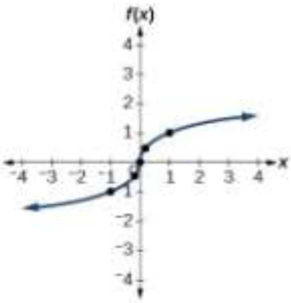


Table 3 Figure 10

8. Cube root Function

A square root function y = √x is a function with the variable under the square root. Similarly, a cube root function is a function with the variable under the cube root. $y = \sqrt[3]{x}$ is the cube root function under the variable x. We can graph these basic functions by finding some points that satisfy each function. The graph of the cubic function is given in Figure 11.



x	f(x)
−1	−1
−0.125	−0.5
0	0
0.125	0.5
1	1

Figure 11 Table 4

9. The Reciprocal Function

An important member of the linear family is the y = x. The graph of this function is a line that passes through the origin, and has a slope of 1. But what does its reciprocal look like? The function y = 1/x has a graph that is very different from a line. First, the domain cannot include 0, as the fraction (1/0) is undefined. The range also does not include 0, as a fraction can only be zero if the numerator is zero and the numerator of y = 1/x is always 1.

In order to understand what these limitations mean for the graph, we will consider function values

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near x = 0 and y = 0. First, consider very small values of x. For example, consider x = 0.01. At the x value, y = 1/x = (1/0.01) = 100. Let's choose an x value even closer to 0, such as x = 0.001. Now we have y = 1/x = (1/0.001) = 1000. As we get closer and closer to x = 0, the function values grow without bound. On the other side of the x-axis, if we choose values closer and closer to x = 0, the function values will approach −∞. We can see this behavior in the graph as a vertical asymptote: the graph is asymptotic to the y-axis.

We can also see in the graph that as x approaches +∞ or −∞, the function values approach 0. The exclusion of y = 0 from the range means that the function is asymptotic to the x-axis (Figure 12).

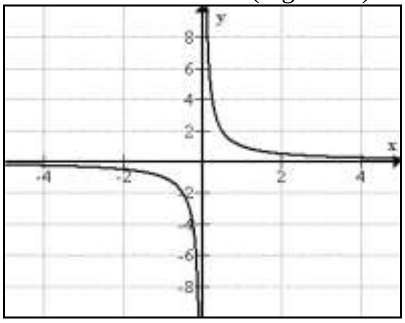

Figure 12

10. Trigonometric Function

The functions which can be expressed by using six trigonometric ratios cos x, sin x, tan x, cot x, cosec x and sec x are called trigonometric functions. Trigonometric functions are also called circular functions and angle functions. In another word, a function such as the sine, cosine, tangent, cotangent, secant, or cosecant of an arc or angle most simply expressed in terms of the ratios of pairs of sides of a right-angled triangle is called circular function. To sketch the graph of the trigonometric function, it is simply a mathematical function of an angle. There are three (sine, cosine, and tangent) basic or primary trigonometric functions and also three (cosecant, secant, and cotangent) inverse functions. The graphs of these functions are given in Figure 13.

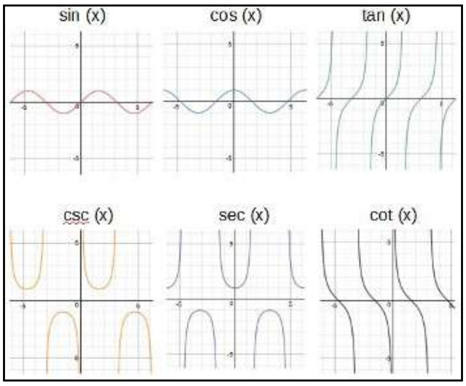

Figure 13

11. Logarithmic Function(Inverse of Exponential Function)

The function is said to be logarithmic if x and y are related such that where a is real constant. The logarithmic function $a^y = x$ is usually written as y = log a^x It means y is the logarithm of x base to a. Since in exponential function, x is acting as an index whereas in logarithmic function y is acting as an index. The inverse of exponential functions $y = a^x$, a>0, a≠1 is $y = \log_a x$ is the logarithmic function. Thus the logarithmic function is the inverse of the exponential function.

Let the inverse of exponential function $y=2^x$ is obtained by switching the x and y coordinates about the line y = x. Recall that if (x, y) is a point on the graph of a function, then (y, x) will be a point on the graph of its inverse. To find the inverse algebraically, begin by interchanging x and y and then try to solve for y. Then the inverse of exponential function $y = 2^x$ is $y = \log_2 x$. which is the logarithmic function. The Table 1 below demonstrates how the x and y values of the points on the exponential curve $y = 2^x$ can be switched to

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find the coordinates of the points on the logarithmic curve $y = \log_2 x$ in Figure 14.

Point on exponential curve	**Point on logarithmic curve**
(-3, 1/8)	(1/8, -3)
(-2, ¼)	(1/4, -2)
(-1, ½)	(1/2, -1)
(0, 1)	(1, 0)
(1, 2)	(2, 1)
(2, 4)	(4, 2)
(3, 8)	(8, 3)

Table 5

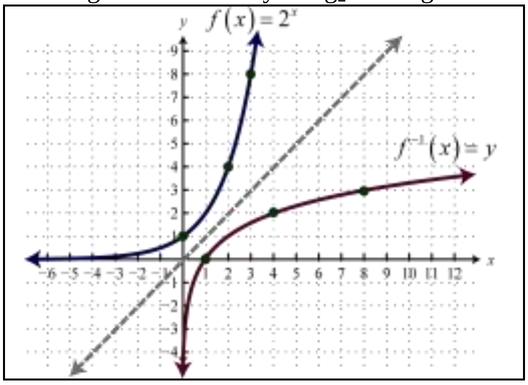

Figure 14

12. Parametric Function

The parametric functions are the functions that express equation of a curve which cannot be represented in the form of a single equation. Generally in physics, such equations are utilized to denote change in an object's position with respect to time. In parametric equations, the two variables say x and y are not expressed in terms of each other, rather x and y are written in terms of another variable, usually t. This variable is known as the parameter. In other words, the parametric functions are the equations which are related to each other by the means of a parameter. Therefore, they are named so. The parametric functions or equations of a curve are expressed in the following general way: $x = f(t)$; $y = \phi(t)$.

The parametric functions are basically used to describe a curve that we cannot define using a general function. It may be a curve that traces around something on itself and we do not have any particular equation for it. So, we use a parameter to represent its curve. It is a little difficult to sketch the graph of the parametric function. The graph of the parametric function $x = 2t + 1$, $y = 4t^2$ is given in Figure 15.

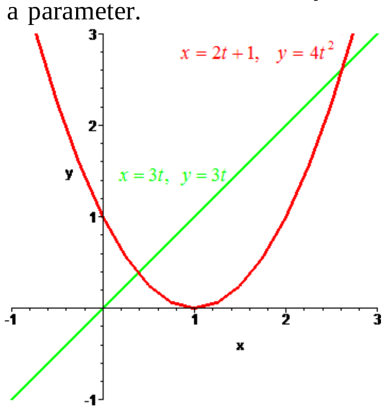

Figure 15

Some Basic Terms for Sketching the Graph of the Function

1. The Domain and Range of the Function: The domain of a function f(x) is the set of all input values (x-values) for the function and the range of a function f(x) is the set of all output values (y-values) for the function. The methods for finding the domain and range vary from problem to problem. The domain of the function is the set of actable values of the variable which is necessary to specify the function. It is not always necessary to specify the domain of the function. i.e. $y = \frac{x^3}{x^3+(1-x)^3}$, $0 \le x \le 1$. It means the domain of the function consists of those x for which $0 \le x \le 1$. If the domain is not specified, we can find the intended domain by assigning the numbers to which the formula defining the function makes

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sense. In the function $y = \frac{1}{x}$, x may be any number except zero.

2. Intercepts and Points: The point of intersection with coordinate axes is called intercept. To find the points where the curve cuts the x-axis, put y = 0 in the equation of the curve. Similarly, to find the points where the curve cuts the y-axis, put x = 0 in the equation of the curve. In the function $y = (x + 1)^2(x - 3)$ if we put y = 0, we get x = -1 & 3and if we put x = 0, then y = -3. So the intercepts are (0, 3), (-1, 0) and (3, 0). This is also known as zeros of the function and we use quadratic and other harder formula for factorization. To find the points in the polar curve, we make a table of r and θ to find their corresponding points.

3. The Slope of a Curve at a Point: The line which touches the curve exactly at a point is called tangent. The slope of the curve at any point say P is also the slope of the tangent line at P. This tangent line at P measures the rate of increase or decrease of the curve as it passes through P. If the equation does not contain any constant term, then the curve passes through the origin. To find the equation of the tangent at the origin is obtained by equating to zero, the lowest degree terms in x and y. i.e. the curve $y^2 = x^2(a^2 - x^2)$ passes through origin. The lowest degree terms are $y^2 = a^2 x^2$. The equations of the tangent at origin are $y = \pm ax$. To find the equation of tangent at any point, we make the first derivative that tends to infinity as the variable x tends to any point. If $f'(x)$ or $\frac{dy}{dx} \to \infty$ as $x \to a$ then x + a will be the equation of tangent at that point.

4. The Derivative: Derivative is a mathematical tool used to measure the rate of change at any function. The graph of the function y = f(x at any point is usually possible to obtain a formula that gives the slope of the curve. This slope formula is called the derivative of the function. Let the linear function $y = mx + c$ where the slope is m. Thus its derivative is m. If the slope of the function is zero, the graph of the function will be a horizontal line. Thus the derivative of a constant function is zero.

The properties of the derivatives $f'(x)$ and $f''(x)$ will determine the properties of the graph of the function f(x). The first derivative of a function f(x) at x = a is positive if $f'(a) > 0$ then f(x) is increasing at x = a. If $f'(a) < 0$ then f(x) is decreasing at x = a. Likewise, if $f''(a) > 0$ than f(x) is concave up at x = a. If $f''(a) < 0$ than f(x) is concave down at x = a.

5. Extreme Point (Relative Extrema): The term relative extrema refers to both relative minimum and relative maximum points on a graph. In other words, an extreme point of a function is a point at which its graph changes from increasing to decreasing and decreasing to increasing. So the minimum and maximum points of the function at the same interval are known as extreme points.

i) A graph has a relative maximum at x = c if f(c) > f(x) for all x in a small enough neighborhood of c.

ii) A graph has a relative minimum at x = c if f(c) < f(x) for all x in a small enough neighborhood of c.

iii) The relative maxima (plural of maximum) and minima (plural of minimum) are the "peaks and valleys" of the graph. There can be many relative maxima and minima in any given graph.

iv) Relative extrema occur at points where f '(x) = 0 or f '(x) does not exist. Use the first derivative test to classify them.

6. Inflection Point or Intervals of Concavity: An inflection point of a graph is a point at which the graph changes from concave up to concave down or vice versa. In another word, concavity is a measure of how curved the graph of the function is at various points. For example, a linear function has zero concavity at all points, because a line simply does not curve. An inflection point of a function f(x) can occur only at a value of x for which $f''(x)$ is zero.

A graph is concave up on an interval if the tangent line falls below the curve at each point in the interval. In other words, the graph curves "upward," away from its tangent lines.

A graph is concave down on an interval if the tangent line falls above the curve at each point in the interval. In other words, the graph curves "downward," away from its tangent lines.

Here's one way to remember the definitions: "Concave up looks like a cup, and concave down looks like a frown."

The second derivative measures concavity:

If $f''(x) > 0$ or $f''(x)$ is positive on an interval, then f is concave up on that interval and If $f''(x) < 0$ or $f''(x)$ is negative on an interval, then f is concave down on that interval.

7. Symmetry: Symmetry is a geometric configuration with respect to a point, a line or a plane if for every point of the configuration there is a corresponding point such that it is the midpoint or the perpendicular bisector of the line segment joining the pairs of points. Such pairs of points are called symmetry. A graph can display various kinds of symmetry. Three main symmetries are especially important: even, odd, and periodic symmetry.

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i) Even symmetry: A function is even if its graph is symmetric by reflection over the y-axis.
ii) Odd symmetry: A function is odd if its graph is symmetric by 180 degree rotation around the origin.
iii) Periodicity: A function is periodic if an only if its values repeat regularly. That is, if there is a value
$p > 0$ such that $f(x + p) = f(x)$ for all x in its domain.

The algebraic test for even/odd is to plug in (-x) into the function.
If $f(-x) = f(x)$, then f is even.
If $f(-x) = -f(x)$, then f is odd.
The conditions for a curve to be symmetry are given below:

i) The curve is symmetrical about the axes if the equation remains unaltered by replacing x by –x and y by -y. The equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is symmetrical about the axes.
ii) The curve is symmetrical about the x -axis if the equation remains the same by replacing x by -x. The equation $y^2 = 4ax$ is symmetrical about x-axis.
iii) The curve is symmetrical about the y -axis if the equation remains the same by replacing y by -y. The equation $y^2 = 4ay$ is symmetrical about y-axis.
iv) The curve is symmetrical about the line y = x if the equation remains the same by interchanging x and y. The equation $x^3 + y^3 = 3axy$ is symmetrical about y = x.
v) The curve is symmetrical in opposite quadrants if the equation remains the same replacing by x = -x and y = -y . The equation xy = c is symmetrical in opposite quadrants.
vi) The curve is symmetrical about the line y = -x if the equation is unaltered by interchanging x and - y.
vii) The curve is symmetrical about the initial line OX if the equation is unaltered by putting $-\theta$ forθ.
viii) The curve is symmetrical about the line OY if the equation of the curve doesn't change by putting $\pi-\theta$ for θ.
ix) The curve is symmetrical about the pole, if the equation contains only the even power of r or the equation doesn't change by putting –r for r

8. Asymptotes: If the perpendicular distance of a straight line from a point on the curve tends to zero as the point moves to infinity along the curve then the straight line is said to be an asymptotes. Here we will discuss only these asymptotes which are necessary to trace the curve. There are three types of asymptotes.

i) A vertical asymptote for a function is a vertical line x = k showing where the function becomes unbounded.
ii) A horizontal asymptote for a function is a horizontal line that the graph of the function approaches as x approaches ∞ or $-\infty$.
iii) An oblique asymptote for a function is a slanted line that the function approaches as x approaches ∞ or $-\infty$.

Both horizontal and oblique asymptotes measure the end behavior of a function. A graph can have an infinite number of vertical asymptotes, but it can only have at most two horizontal asymptotes. Horizontal asymptotes describe the left and right-hand behavior of the graph. A graph will (almost) never touch a vertical asymptote; however, a graph may cross a horizontal asymptote.
The conditions for the curve having asymptotes parallel to x -axis and y -axis are given below.

i) If x = a be an asymptote to the curve y = f(x) parallel to y-axis, y should tend to infinity as x tends to a. i.e. $lt_{x\to a}\ y = \infty$.. Here x = a is an asymptote parallel to y-axis.
ii) If y = b be an asymptote to the curve y = f(x) parallel to x-axis, then $y \to b$ as $x\to\infty$. i.e $lt_{x\to\infty}\ y = b$.
y = b is an asymptote parallel to x-axis.
iii) If the equation is of degree n and the term x^n is absent then the asymptote parallel to x-axis is obtained by equating to zero the coefficient of x^{n-1} If it is also absent then equate the coefficient of lower degree preceding terms to zero.
iv) If the equation is of degree n and the term y^n is absent, then the asymptote parallel to y-axis is obtained by equating to zero to the coefficient of lower degree preceding terms. i.e. $x^2y^2 - x^2y - xy^2 + x + y + 1 = 0$. The curve is of four degree then $y^2 - y = 0$ & $x^2 - x = 0$ are the asymptotes parallel to x-axis and y-axis respectively.

9. Region: The entire area where the curve lies is simply the region. The region can generally be studied from the equation of the curve. Some possible rules to find the region are given below:

i) If y becomes imaginary when x is greater than a certain number a then no part of the curve lies to the right side of the line x = a Likewise when x is less than a certain number a and if y becomes imaginary then no part of the curve lies to the left of the line x = a.

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ii) When y is greater than any number c and if x becomes imaginary then no part of the curve lies on the line y = c.

iii) If r becomes imaginary by the corresponding value of θ then no part of the curve lies in that portion. In the function $r^2 = a^2 \cos 2\theta$ when the value of θ lies between $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ then r becomes imaginary. So, no part of the curve lies between $\theta = \frac{\pi}{4}$ and $\theta = \frac{3\pi}{4}$.

iv) If x becomes imaginary when y is less than any number c then no part of the curve lies below the line y = c In the function y = 4ax, y becomes imaginary when x is negative. So no part of the curve lies in the left of the y-axis.

Steps for Sketching the Graph of the Cartesian Function

1. Compute f'(x) and f″(x).
2. Find the maximum and minimum points.

a. Put f'(x) = 0 and solve for x.

i) If f″(x) > 0, the curve is concave up at x = a then f(x) has a minimum point at x = a The minimum point in the function is (a, f (a))

ii) If f″(a) < 0, the curve is concave down at x = a, so f(x) has a maximum point at x = a The maximum point is (a, f (a))

iii) If f″(a) = 0; examine f '(x) to the left and right of x = a in order to determine if the function changes from increasing to decreasing or vice versa.

b. Make a partial sketch of the graph near to each point where f(x) has a horizontal tangent line.

3. Determine the concavity of f(x).

a. Set f″(x) = 0 and solve for x. Suppose that f″(b) = 0.Next test the concavity for x near to b. If the concavity changes at x = b, then (b, f (b)) is an inflection point. Otherwise, the concavity at x = b is the same as other nearby points.

4. Consider other properties of the function and complete the sketch.

a. If f(x) is defined at x = 0 then y-intercept is (0, f (0)).

b. Does the partial sketch suggest that there is x-intercept? If so, it can be found by setting f(x) = 0and solve for x.

c. Observe where f(x) is defined. Sometimes the function is given only for restricted value of x. Sometimes the formula for f(x) is meaningless for certain value of x.

5. Check for possible asymptotes.
6. Check the function either it is symmetrical or not.
7. Determine the region.
8. Complete the sketch.

Steps for Sketching Graph of the Polar Function

i) If possible, change the polar equation into Cartesian and follow the above steps.

ii) Determine the curve whether it is symmetrical or not.

iii) Find the corresponding values of r and θ

iv) Determine the region.

v) Determine the tangent at the pole.

vi) Determine the intersection of the curve with the initial line OX and OY.

vii) Find the value of r for any real value of θ ; if r = 0 for any real value of θ , the curve passes through the origin.

viii) Complete the sketch.

Steps for Sketching the Graph of the Parametric Function

i) If possible, change the parametric function into Cartesian form and follow the above steps of Cartesian form. If it is not possible then find $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$.

ii) Find the corresponding value of x, y, $\frac{dy}{dx}$ and Ψfor different values of t.

iii) Plot the different values of x and y and find out the slopes of the tangents i.e. $\frac{dy}{dx}$ at these points.

iv) Find the region.

v) Check either the curve is symmetrical or not by putting t by –t. If it is equal then the curve is symmetrical.

vi) Complete the sketch.

Some Examples of Sketching the Graph of the Function

Example -1: Sketch the graph of: $y = x^3 - 3x^2 + 5$

Solution: $f'(x) = 3x^2 - 6x$. Put f′(x) = 0 then we get x = 0 & 2. Therefore f(0) = 5 & f(2) = 1.Again, f″(x) = 6x – 6 then f″(0) = –6 & f″(2) = 6.

Since f″(0) is negative then the graph is concave down at $x = 0$ and f″(2) is positive then the graph is concave up at x = 2.Hence (0,5) is maximum point and (2,1) is maximum point. The partial sketch is as shown in Figure-15.

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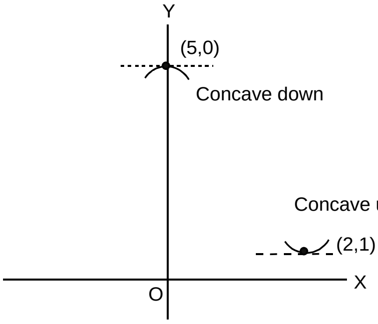

Figure-15a

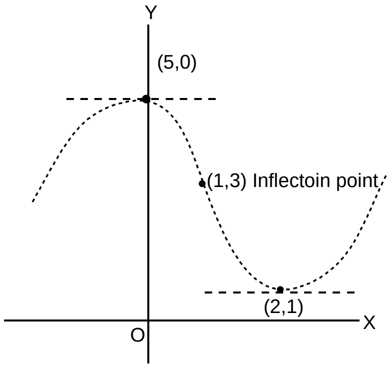

Figure-15b

To find the inflection point, put $f''(x) = 0$ then we get $x = 1$ and $f(1) = 3$, that is (1, 3). Joining these points we can complete the sketch as in the above Figure 15b.

Example-2: Sketch the graph of: $y^2 = \frac{x^2(a+x)}{(a-x)}$ (Strophoid).

Solution:

i) The curve is symmetrical about x-axis (absence of y^3)

ii) The curve passes through (0, 0) and (-a, 0).(absence of constant term)

iii) Tangents at origin are $y = \pm x$. (coefficient of lowest degree term equated to zero)

iv) Tangents at $(-a, 0)$ is $x = -a$. ($y \to \infty$ as $x \to a$).

v) The asymptote parallel to y-axis is $x = a$. ($y \to \infty$ as $x \to a$).

vi) y becomes imaginary if $x > a$ or $x < -a$.

The graph of the function is given in Figure 16.

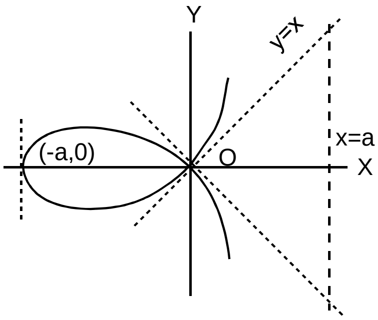

Figure 16

Example-3: Sketch the graph of: $r^2 = a^2 \sin\theta$.

Solution: Now changing into Cartesian form, $r^2 = a^2 \sin\theta \Rightarrow x^2 + y^2 = 2a^2xy$ then,

i) The curve is symmetrical about the line $y = x$.

ii) The curve passes through origin.

iii) Table of r and θ.

The graph of the function is given in Figure 17.

$\theta = 0^0$	45^0	90^0
$r = 0$	$\pm a$	0

Table 3

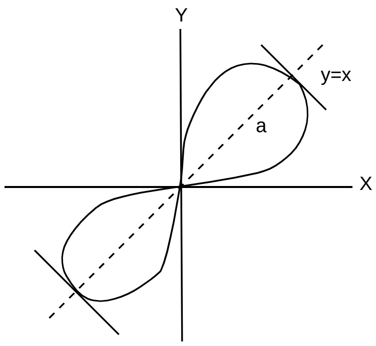

Figure 17

Some Real Life Application of the functions

i) Many real world situations can be modeled by functions.
ii) Applications of linear functions are estimating the adult height, loss of population, total cost of any product or services etc.
iii) Quadratic function helps to maximize and minimize the value. i.e. Maximizing area and volume, minimizing cost, maximizing profit, finding height of the cliff, finding depth of the well etc.
iv) Most frequent application of exponential functions is to find the interest compounded annually.
v) Rational function can be applicable to estimate temperature during illness, to find the average cost of any company product (cost benefit model), population growth in the certain time, determine the total profit of the product, height of the thrown object, estimate medical dosage, height of a launched rocket, co relational relation: like cholesterol level and the risk of heart attack etc.
vi) The most important application of logarithmic function is to find walking speed of a person in a certain time, loudness of sound, rate of forgetting of the student in a certain time, spread rate of an epidemic, population growth, radioactive decay, effect of advertising, supply and demand, growth rate and measurement of earthquake magnitude etc.
vii) The application of trigonometric function is to find the height of any object like cloud in the sky, sound of an airplane, satellite location etc.

III. CONCLUSIONS

In the real world situation, it's very common that one quantity depends on another quantity called variables and we observe there is a relationship between them. If we find for every value of the first variable there is only one value of the second variable, then we say the second variable is a function of the first variable. The first variable is the independent variable which is usually written as x and the second variable is the dependent variable written as y. It is a rule that relates how one quantity depends on other quantities. So, function is one or more rules that are applied to an input and yield an output. The input is the number or value put into a function. The output is the number or value the function gives out. The graph of a function is the set of all points whose co-ordinates (x, y) satisfy the function y = f(x). This means that for each x-value there is a corresponding y-value which is obtained when we substitute into the given function.

Functions are essential in every field of applied mathematics as well as real life situation. They are useful to the statistician to test the significant difference of the product or to find the observation is significant or not. It is necessary to the climatologist for estimating the temperature. Government accountants need the function to estimate the revenue under the current tax policy. Similarly, the computer programmer relies on functional language of programming as well as a graphics programmer need the function for making code. The graph of a function is really useful to model a real-world problem of different sector. Sometimes we may not know an expression for a function but we do know some values. The graph can give us a good idea of what function may be applied to the situation to solve the problem.

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