

# Harmonics Reduction with Active Filter p-q Theory Control in Three Phase Power System

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## I. INTRODUCTION

Due the intensive use of power converters and other non-linear loads in industry and by consumers in general, it can be observed an increasing deterioration of the power systems voltage and current waveforms. The presence of harmonics in the power lines results in greater power losses in distribution, interference problems in communication systems and, sometimes, in operation failures of electronic equipments, which are more and more sensitive since they include

microelectronic control systems, which work with very low energy levels. Because of these problems, the issue of the power quality delivered to the end consumers is, more than ever, an object of great concern.

### Active Filters

There are basically two types of active filters: the shunt type and the series type. It is possible to find active filters combined with passive filters as well as active filters of both types acting together.

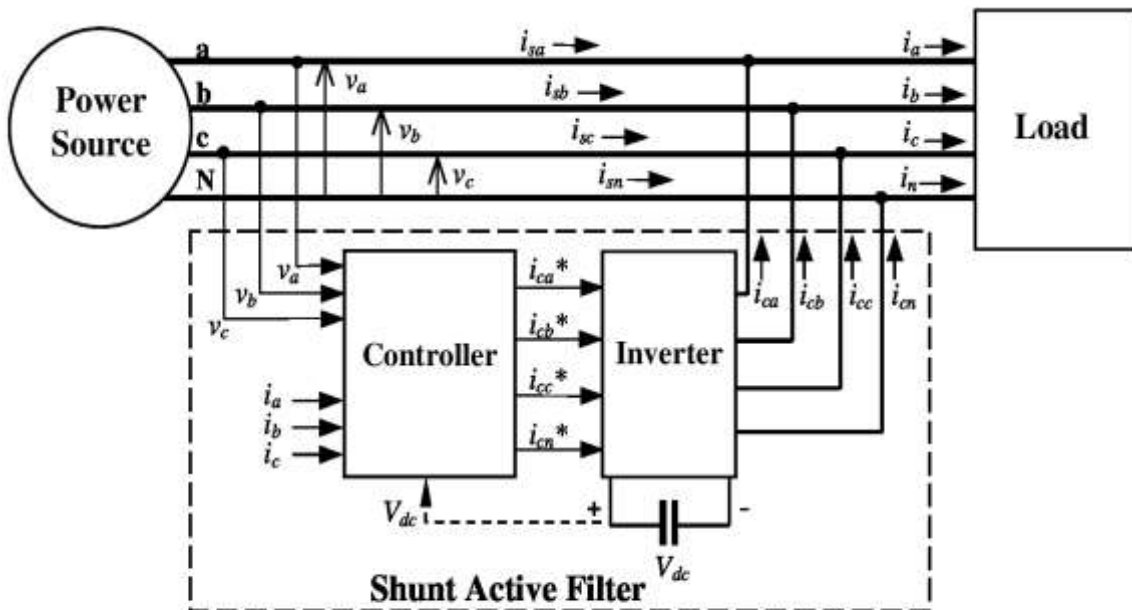


Fig. 1 - Shunt active filter in a three-phase power system.

Fig. 1 presents the electrical scheme of a shunt active filter for a three-phase power system with neutral wire, which is able to compensate for both current harmonics and power factor. Furthermore, it allows load balancing, eliminating the current in the neutral wire. The power stage is, basically, a voltage-source inverter with only a single capacitor in the DC side (the active filter

does not require any internal power supply), controlled in a way that it acts like a current-source. From the measured values of the phase voltages ( $v_a$ ,  $v_b$ ,  $v_c$ ) and load currents ( $i_a$ ,  $i_b$ ,  $i_c$ ), the controller calculates the reference currents ( $i_{ca}^*$ ,  $i_{cb}^*$ ,  $i_{cc}^*$ ,  $i_{cn}^*$ ) used by the inverter to produce the compensation currents ( $i_{ca}$ ,  $i_{cb}$ ,  $i_{cc}$ ,  $i_{cn}$ ).

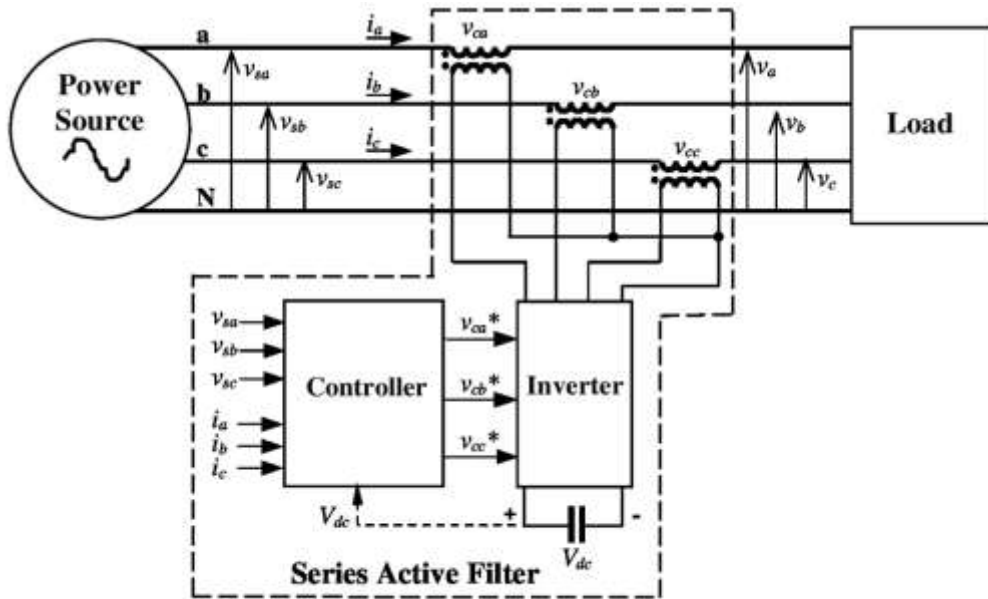


Fig. 2 - Series active filter in a three-phase power system.

Fig-2 shows the scheme of a series active filter for a three-phase power system. It is the dual of the shunt active filter, and is able to compensate for distortion in the power line voltages, making the voltages applied to the load sinusoidal. The filter consists of a voltage-source inverter

(behaving as a controlled voltage source) and requires 3 single-phase transformers to interface with the power system. The series active filter does not compensate for load current harmonics but it acts as high-impedance to the current harmonics coming from the power source side.

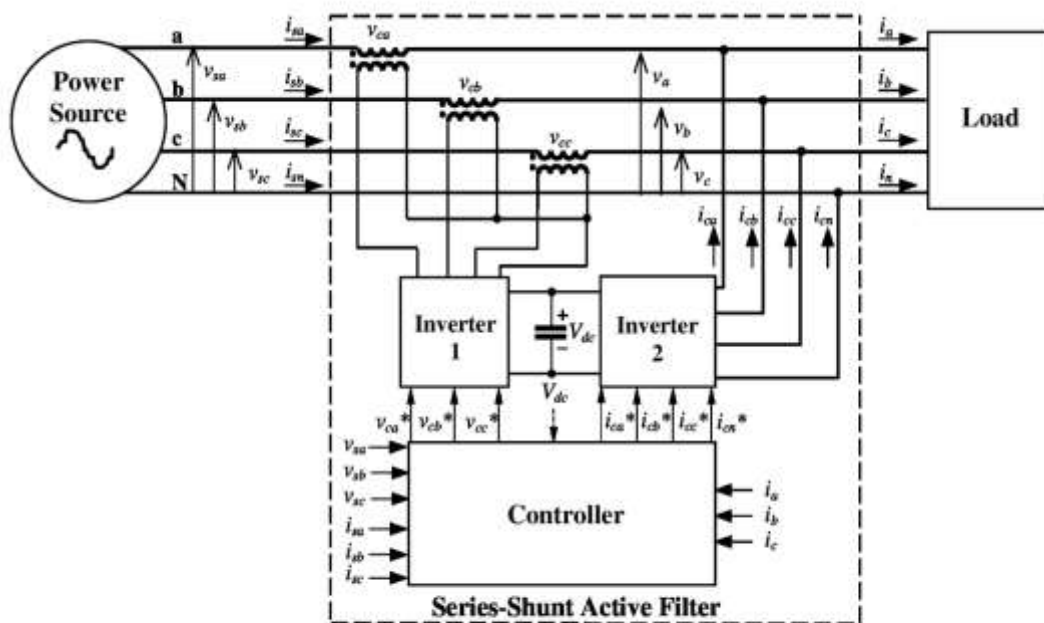


Fig. 3 - Series-shunt active filter in a three-phase power system.

**The p-q theory**

"The Generalized Theory of the Instantaneous Reactive Power in Three-Phase Circuits", also known as instantaneous power theory, or p-q theory. It is based on instantaneous values in three-phase power systems with or without neutral wire, and is valid for steady-state or

transitory operations, as well as for generic voltage and current waveforms. The p-q theory consists of an algebraic transformation (Clarke transformation) of the three-phase voltages and currents in the a-b-c coordinates to the  $\alpha$ - $\beta$ -0 coordinates, followed by the calculation of the p-q theory instantaneous power components:

$$\begin{bmatrix} v_0 \\ v_\alpha \\ v_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \cdot \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{1}$$

$$p_0 = v_0 \cdot i_0 \quad \text{instantaneous zero-sequence power} \tag{2}$$

$$p = v_\alpha \cdot i_\alpha + v_\beta \cdot i_\beta \quad \text{instantaneous real power} \tag{3}$$

$$q = v_\alpha \cdot i_\beta - v_\beta \cdot i_\alpha \quad \text{instantaneous imaginary power (by definition)} \tag{4}$$

The power components  $p$  and  $q$  are related to the same  $\alpha$ - $\beta$  voltages and currents, and can be written together:

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} v_\alpha & v_\beta \\ -v_\beta & v_\alpha \end{bmatrix} \cdot \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \tag{5}$$

These quantities are illustrated in Fig. 4 for an electrical system represented in a-b-c coordinates and have the following physical meaning:

$p_0$  = mean value of the instantaneous zero-sequence power – corresponds to the energy per time unity which is transferred from the power supply to the load through the zero-sequence components of voltage and current.

$p \sim$  = alternated value of the instantaneous real power – It is the energy per time unity that is exchanged between the power supply and the load, through the a-b-c coordinates.

$q$  = instantaneous imaginary power – corresponds to the power that is exchanged between the phases of the load. This component does not imply any transference or exchange of energy between the power supply and the load, but is responsible for the existence of undesirable currents, which circulate between the system phases. In the case of a balanced sinusoidal voltage supply and a balanced load, with or without harmonics,  $q$  (the mean value of the instantaneous imaginary power) is equal to the conventional reactive power ( $3 \cdot I \cdot \phi_1$   $q = \cdot V \cdot I \cdot \sin$ ).

**The p-q theory applied to shunt active filters**

The p-q theory is one of several methods that can be used in the control active filters [3-11]. It presents some interesting features, namely: -

- It is inherently a three-phase system theory; -
- It can be applied to any three-phase system (balanced or unbalanced, with or without harmonics in both voltages and currents); -
- It is based in instantaneous values, allowing excellent dynamic response; -
- Its calculations are relatively simple (it only includes algebraic expressions that can be implemented using standard processors); -
- It allows two control strategies: constant instantaneous supply power and sinusoidal supply current.

It is also possible to conclude from Fig. 5 that the active filter capacitor is only necessary to compensate  $p \sim$  and  $p_0 \sim$ , since these quantities must be stored in this component at one moment to be later delivered to the load. The instantaneous imaginary power ( $q$ ), which includes the conventional reactive power, is compensated without the contribution of the capacitor. This means that, the size of the capacitor does not depend on the amount of reactive power to be compensated.

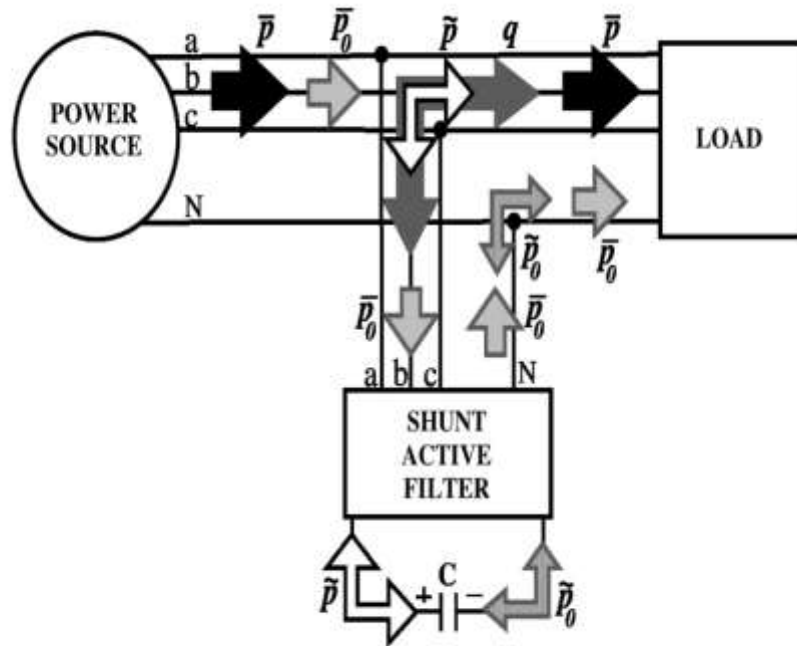


Fig. 5 - Compensation of power components  $\tilde{p}$ ,  $q$ ,  $\tilde{p}_0$  and  $\bar{p}_0$  in  $a$ - $b$ - $c$  coordinates.

To calculate the reference compensation currents in the  $\alpha$ - $\beta$  coordinates, the expression (5) is inverted, and the powers to be compensated ( $\tilde{p}$ ,  $\tilde{p}_0$  and  $q$ ) are used:

$$\begin{bmatrix} i_{ca}^* \\ i_{cb}^* \end{bmatrix} = \frac{1}{v_\alpha^2 + v_\beta^2} \begin{bmatrix} v_\alpha & -v_\beta \\ v_\beta & v_\alpha \end{bmatrix} \begin{bmatrix} \tilde{p} - \tilde{p}_0 \\ q \end{bmatrix} \quad (6)$$

Since the zero-sequence current must be compensated, the reference compensation current in the 0 coordinate is  $i_0$  itself

$$i_{c0}^* = i_0 \quad (7)$$

In order to obtain the reference compensation currents in the  $a$ - $b$ - $c$  coordinates the inverse of the transformation given in expression (1) is applied:

$$\begin{bmatrix} i_{ca}^* \\ i_{cb}^* \\ i_{cc}^* \end{bmatrix} = \frac{\sqrt{2}}{\sqrt{3}} \begin{bmatrix} 1/\sqrt{2} & 1 & 0 \\ 1/\sqrt{2} & -1/2 & \sqrt{3}/2 \\ 1/\sqrt{2} & -1/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} i_{c0}^* \\ i_{c\alpha}^* \\ i_{c\beta}^* \end{bmatrix}$$

$$i_{cn}^* = -(i_{ca}^* + i_{cb}^* + i_{cc}^*) \quad (8)$$

- the phase supply currents become sinusoidal, balanced, and in phase with the voltages.  
 (in other words, the power supply “sees” the load as a purely resistive symmetrical load);

-the neutral current is made equal to zero (even 3rd order current harmonics are compensated);  
 - the total instantaneous power supplied

$$p_{3s}(t) = v_a \cdot i_{sa} + v_b \cdot i_{sb} + v_c \cdot i_{sc} \quad (9)$$

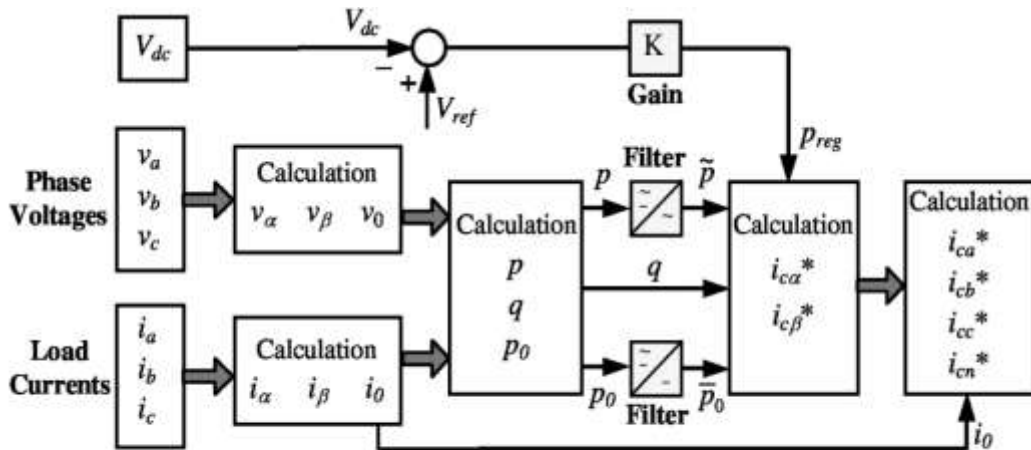


Fig. 6 – Calculations for the constant instantaneous supply power control strategy.

The sinusoidal supply current control strategy must be used when the voltages are distorted or unbalanced and sinusoidal currents are desired. The block diagram of Fig. 7 presents the calculations required in this case.

- the phase supply currents become sinusoidal, balanced, and in phase with the fundamental

voltages;

- the neutral current is made equal to zero (even 3rd order current harmonics are compensated);  
 - the total instantaneous power supplied ( p<sub>3S</sub> ) is not made constant, but it presents only a small ripple (much smaller than before compensation)

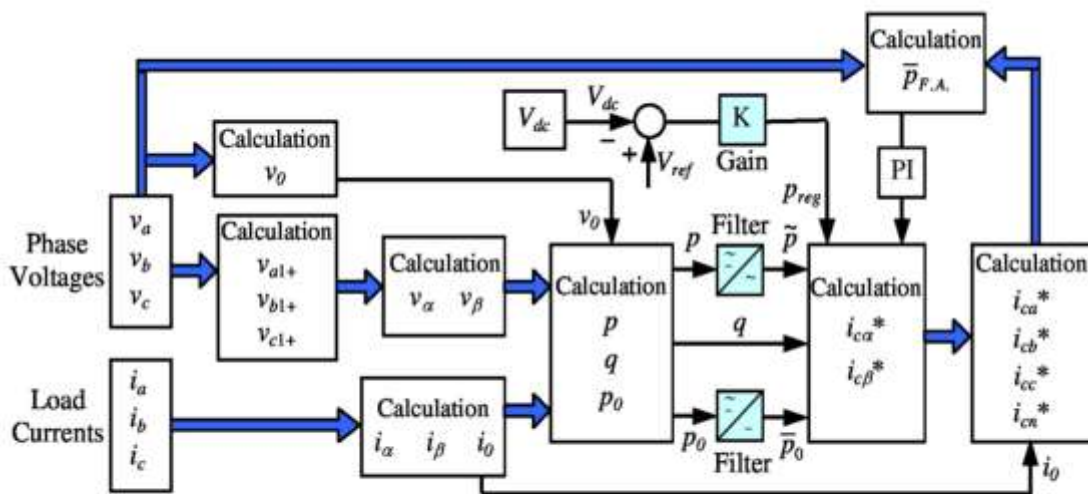


Fig. 7 – Calculations for the sinusoidal supply current control strategy.



The practical implementation of the shunt active filter demands the regulation of the voltage at the inverter DC side (Vdc - the capacitor voltage) as suggested in Fig. 6 and Fig. 7, where Vref is the reference value required for proper operation of the active filter inverter.

Figures 8, 9 and 10, present simulation results using Matlab/Simulink [14, 15] for a three-phase power system with a shunt active filter. They

include the following waveforms, corresponding to two-cycles of steady-state operation: total instantaneous power at load and source, phase voltages, load and source currents (phase and neutral currents). In the cases with distorted voltages the voltage total harmonic distortion (THD) is equal to 10%, which is a higher value than what is regulated by any power quality standard.

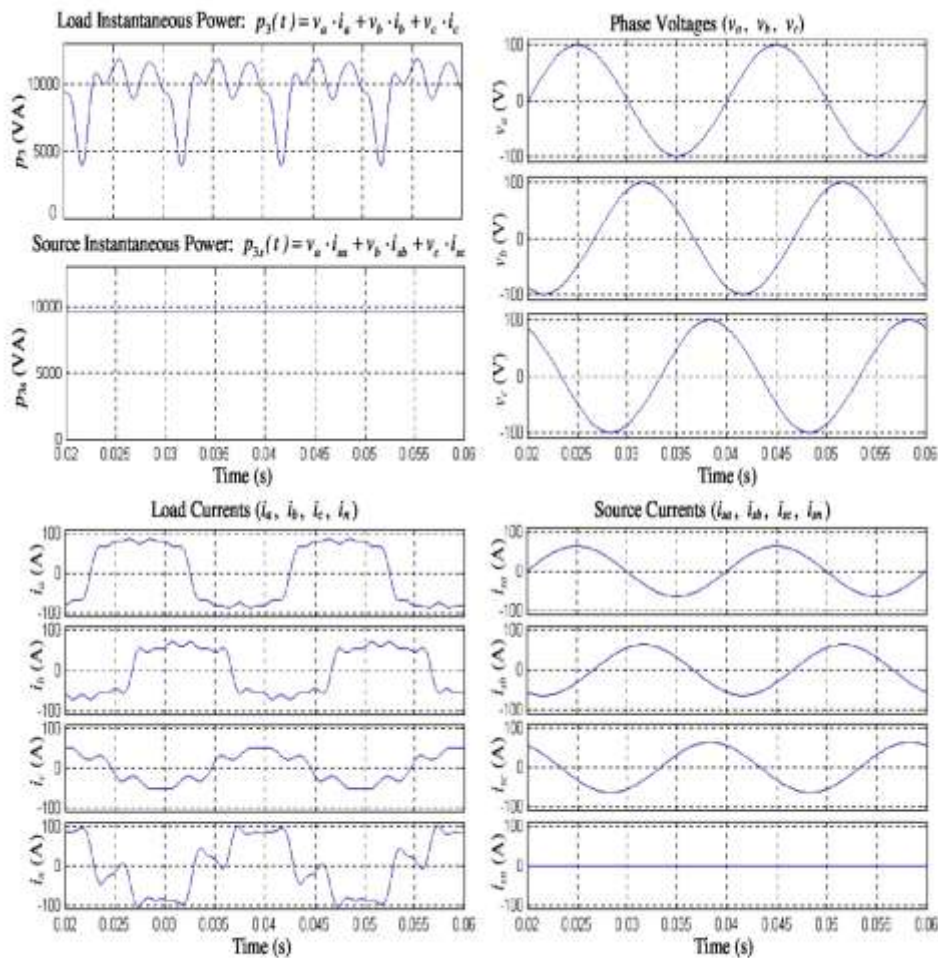


Fig. 8 – Simulation results for the constant instantaneous supply power strategy with sinusoidal voltages.

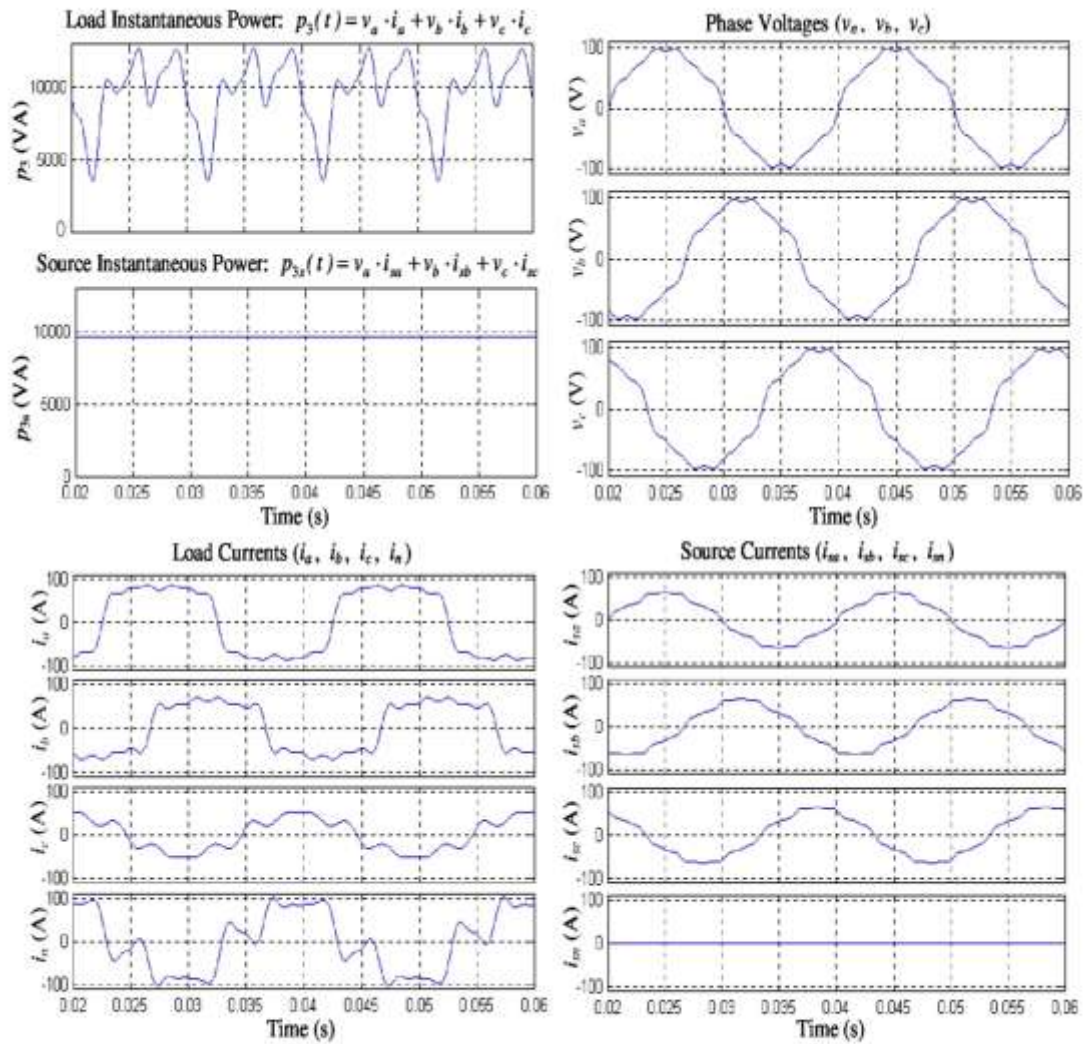


Fig. 9 - Simulation results for the constant instantaneous supply power strategy with distorted voltages.

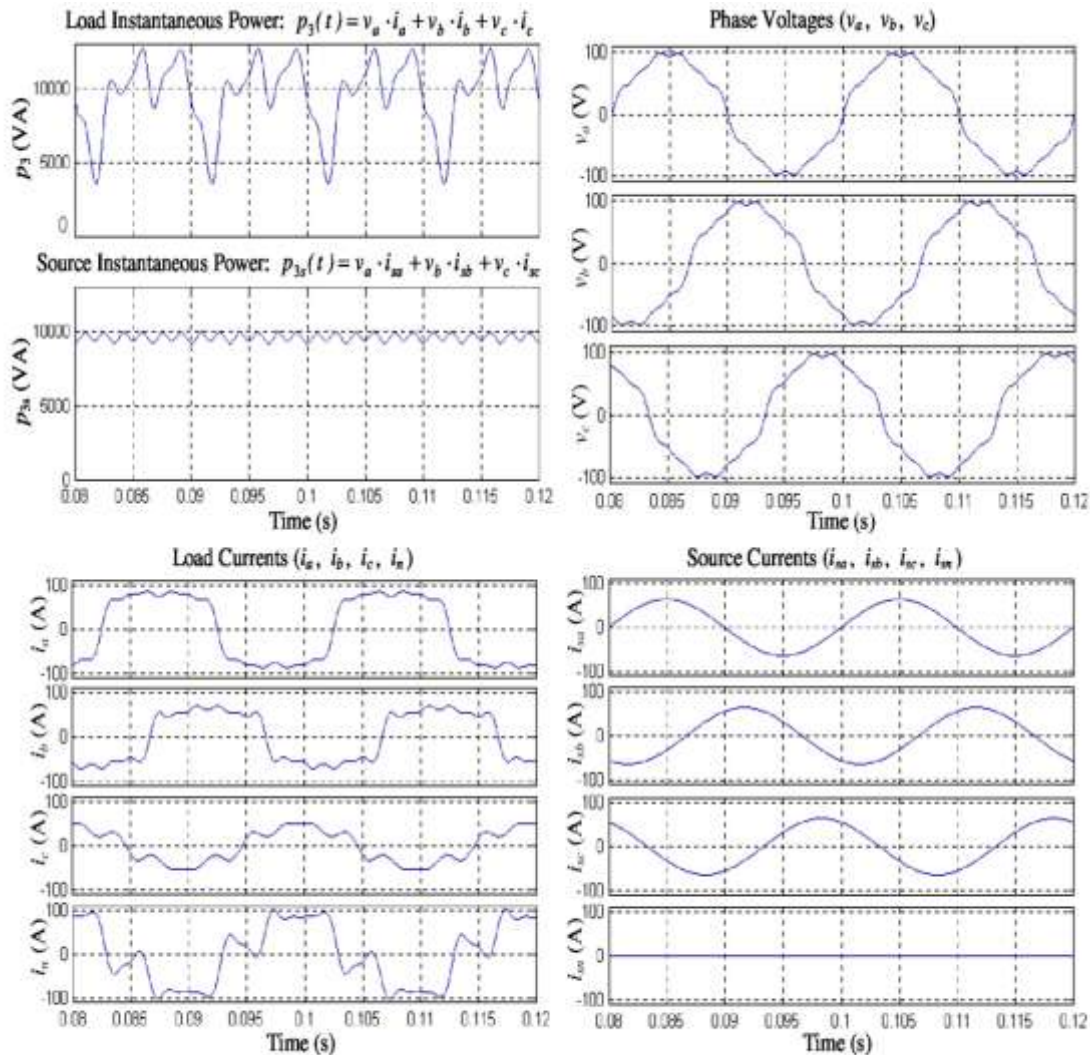


Fig. 10 - Simulation results for the sinusoidal supply current strategy with distorted voltages.

## II. CONCLUSIONS

Active filters are an up-to-date solution to power quality problems. Shunt active filters allow the compensation of current harmonics and unbalance, together with power factor correction, and can be a much better solution than the conventional approach (capacitors for power factor correction and passive filters to compensate for current harmonics).

This paper presents the p-q theory as a suitable tool to the analysis of non-linear three-phase systems and for the control of active filters.

Based on this theory, two control strategies for shunt active filters were described, one leading to constant instantaneous supply power and the other to sinusoidal supply current.

The implementation of active filters based on the p-q theory are cost-effective solutions,

allowing the use of a large number of low-power active filters in the same facility, close to each problematic load (or group of loads), avoiding the circulation of current harmonics, reactive currents and neutral currents through the facility power lines.

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