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Expressing the Factorial of a number as the Linear Combination of Perfect Squares

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ABSTRACT: This article is published on relationship between the Factorial and the perfect squares, where the Factorial of a number can be expressed as the Linear combination of Perfect Squares and the coefficients of perfect Squares are either +1 or -1. Factorial is the product of some of the first finite Natural Numbers (say n). However such a Linear Combination may or may not be unique.

Key Words: Factorial, Natural Number, Perfect Squares, Coefficients

Introduction: This article which is going to be published, can show that how the Factorial of a number can be uniquelyexpressed as the Linear Combination of Perfect Squares. Where the coefficients of Prefect Squares are either +1 or -1. All such Prefect Squares are not equal to each other. This concept is Valid for all Natural numbers including zero. The factorial of a Number is the summation of some of the finite number of positive and negative perfect squares. But we can give a general formula to express the relationship between the factorial and the summation:

$n! = \sum_{i=1}^{k} \pm n_i^2$ where i=1,2,3,…..,k

However, for any further enquiry the books mentioned below can be checked for references.

Theorem: Every factorial can express as the Linear Combination of Perfect squares.

Proof: To prove the above theorem we need to show it with the help of certain examples:

We know, $0! = 1 = 1^2$

Again, $1! = 1 = 1^2$

Also, $2! = 2 = 36-25-9 = 6^2-5^2-3^2 = 6^2 + (-5^2) + (-3^2)$

$2! = 6^2 + (-5^2) + (-3^2)$

Again, $3! = 6 = 5+1 = 9-4+1 = 3^2-2^2+1^2 = 3^2+(-2^2)+1^2$

So, $3! = 3^2 + (-2^2) + 1^2$

Similarly, we can give certain examples further such as:

$4! = 24 = 5^2 + (-1^2)$

$5! = 120 = 11^2 + (-1^2)$

$6! = 720 = 27^2 + (-3^2)$

$7! = 5040 = 70^2 + 12^2 + (-2^2)$

$8! = 40320 = 200^2 + 17^2 + 16^2 + (-15^2)$

$9! = 362880 = 600^2 + 50^2 + 18^2 + 8^2 + 4^2 + (-5^2) + 1^2$

$10! = 3628800 = 1900^2 + 120^2 + 70^2 + (-20^2) + (-10^2)$

From the above few expressions we can see that factorial of a number can be expressed as the Linear combination of Perfect squares, where the coefficients of perfect squares are either +1 or -1. Continuing in this manner for any such larger value of Natural Numbers, we would definitely get the required format.

The factorial of the number ‘n’ is the summation of positive and negative perfect squares. Thus there exists a common formula for the above expressions, which is given by:

$n! = \sum_{i=1}^{k} \pm n_i^2$ where, I = 1, 2, 3,… k where ‘k’ is any finite value.

CONCLUSION:From the above proof of the theorem it can be concluded that, for every ‘n’ belonging to the set of Natural Numbers including ‘zero’ can be expressed as the finite Linear Combination of Perfect squares.

Books for references: There are certain books which are listed below:

- An introduction to the theory of Numbers by G.H. Hardy
- An Introduction to Linear Algebra by V. Krishnamurthy, Mania.
- Introduction to Linear Algebra by David .C.Lay
- Concepts of Linear Algebra by Pramod Kumar Saikiya.