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Exact Solution for a Nonlinear Equation

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ABSTRACT: In this paper, we derive exact traveling wave solutions ofnonlinear D-S conduction equation by a presented method.The method appears to be efficient in seeking exact solutionsof nonlinear equations.

KEYWORDS:(G'/G)-expansion method, traveling wave solutions,exact solution, evolution equation, nonlinear D-S equation.

I. INTRODUCTION

In scientific research, seeking the exact solutions of nonlinearequations is a hot topic. Many approaches have beenpresented so far [1-6]. In [7], Mingliang Wang propo-seda new method called (G'/G)-expansion method. The mainmerits of the (G'/G)-expansion method over the other methodsare that it gives more general solutions with some freeparameters and it handles NLEEs in a direct manner withno requirement for initial/boundary condition or initial trialfunction at the outset. So the application of the (G'/G)-expansion method attracts many author's attention.Our aim in this paper is to present an application ofthe (G'/G)-exp-ansion method to nonlinear D-S equation.

II. DESCRIPTION OF THE(G'/G)-EXPANSION METHOD

In this section we will describe the (G'/G)-expansion methodfor finding out the traveling wave solutions of no-nlinear evolution equations.

Suppose that a nonlinear equation, say in three indep-endentvariables x, y and t , is given by

$$P(u,u_{t,}u_x,u_y,u_{tt},u_{xt},u_{yt},u_{xx},u_{yy}......)=0$$

(2.1)

where u = u(x, y, t) is an unknown function,P isa polyno-mial in u = u(x, y, t) and its various partialderivatives, in which the highest order derivatives andnonlinear terms are involved. In the following we give themain steps of the (G'/G)-expansion method.

Step 1. Combining the independent variables x, y and t into one variable $\xi=\xi(x,y,t)$, we suppose that

$$u(x,y,t)=u(\xi),\xi=\xi(x,y,t)$$

(2.2)

the travelling wave variable (2.2) permits us reducing Eq.

(2.1) to an ODE for $u=u(\xi)$

$$P(u,u',u'',......)=0 \quad (2.3)$$

Step 2. Suppose that the solution of (2.3) can be expre-ssedby a polynomial in (G'/G) as follows:

$$u(\xi)=\alpha_m(\tfrac{G'}{G})^m+...... \quad (2.4)$$

where $G=G(\xi)$ satisfies the second order LODE in theform

$$G''+\lambda G'+\mu G=0 \quad (2.5)$$

$\alpha_m,...\lambda$ and μ are constants to be determined later, $\alpha_m\neq 0$.The unwritten part in (2.4) is also a polynomial in $(\frac{G'}{G})$, the degree of which is generally equal to or less than m−1.The positive integer m can be determined by consider-ing thehomogeneous balance between the highest order de-rivativesand nonlinear terms appearing in (2.3).

Step 3. Substituting (2.4) into (2.3) and using second
order LODE (2.5), collecting all terms with the sameorder of $(\frac{G'}{G})$ together, the left-hand side of Eq. (2.3)is converted into another polynomial in $(\frac{G'}{G})$. Equatingeach coefficient of this polynomial to zero, yields a set ofalgebraic equation-ns for $\alpha_m,...\lambda$ and μ.

Step 4. Assuming that the constants $\alpha_m,...\lambda$ and μ canbe obtained by solving the algebraic equations in Step 3,since the general solutions of the second order LODE (2.5)

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have been well known for us, substituting $\alpha_m,\ldots$ and thegeneral solutions of Eq. (2.5) into (2.4) we can obtain thetraveling wave solutions of the nonlinear evolution equation(2.1).

In the subsequent sections we will illustrate the propo-sedmethod in detail by applying it to a nonlinear evolutionequation.

III. APPLICATION OF (G'/G)-EXPANSION METHOD FORNONLINEAR D-S EQUATION

In this section, we will consider the following nonline-ar D-S equation:

$$u_t+(v^2)_x=0 \quad (3.1)$$

$$v_t-v_{xxx}+3vu_x+3uv_x=0 \quad (3.2)$$

Supposing that

$$\xi=kx-\omega t \quad (3.3)$$

By (3.3), (3.1) and (3.2) are converted into ODEs

$$-\omega u'+k(v^2)'=0 \quad (3.4)$$

$$-\omega v'-k^3v'''+3kvu'+3kuv'=0 \quad (3.5)$$

Integrating (3.4) and (3.5) once, we have

$$-\omega u+kv^2=g_1 \quad (3.6)$$

$$-\omega v-k^3v''+3kuv=g_2 \quad (3.7)$$

Suppose that the solution of (3.6) and (3.7) can be expressed by apolynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi)=\sum_{i=0}^{m}a_i(\tfrac{G'}{G})^i \quad (3.8)$$

$$v(\xi)=\sum_{i=0}^{n}b_i(\tfrac{G'}{G})^i \quad (3.9)$$

where a_i, b_i are constants, $G=G(\xi)$ satisfies the second orderLODE in the form:

$$G''+\lambda G'+\mu G=0 \quad (3.10)$$

where λ and μ are constants.

Balancing the order of u and v^2 in Eq.(3.6),the order of v'' and uv in Eq.(3.7), we can obtain $m=2n, m+2=m+n \Rightarrow m=2, n=1$.

So Eq.(3.8) and (3.9) can berewritten as

$$u(\xi)=a_2(\tfrac{G'}{G})^2+a_1(\tfrac{G'}{G})^1+a_0, a_2\neq 0$$
(3.11)

$$v(\xi)=b_1(\tfrac{G'}{G})^1+b_0, b_1\neq 0$$
(3.12)

a_2,a_1,a_0,b_1,b_0 are constants to be determined later.

Substituting (3.11) and (3.12) into (3.6) and (3.7) and collecting all the termswith the same power of $(\frac{G'}{G})$ toge-ther and equating eachcoefficient to zero, yields a set of simultaneous algebraicequations as follows:

For Eq.(3.6):

$$(\frac{G'}{G})^0: -\omega a_0-g_1+kb_0^2=0$$

$$(\frac{G'}{G})^1: -\omega a_1+2kb_1b_0=0$$

$$(\frac{G'}{G})^2: kb_1^2-\omega a_2=0$$

For Eq.(3.7):

$$(\frac{G'}{G})^0: -\omega b_0-g_2-k^3b_1\lambda\mu+3ka_0b_0=0$$

$$(\frac{G'}{G})^1: -k^3b_1\lambda^2+3ka_0b_1-\omega b_1-2k^3b_1\mu+3kb_0a_1=0$$

$$(\frac{G'}{G})^2: 3ka_1b_1-3k^3b_1\lambda+3kb_0a_2=0$$

$$(\frac{G'}{G})^3: -2k^3b_1+3kb_1a_2=0$$

Solving the algebraic equations above, yields:

$$a_2=\frac{2}{3}k^2, a_1=\frac{2}{3}k^2\lambda,$$

$$a_0=\frac{1}{6}\frac{3b_1^2+4k^4\mu}{k^2}, b_1=b_1, b_0=\frac{1}{2}b_1\lambda$$

$$k=k, \omega=\frac{3b_1^2}{2k}, g_1=\frac{b_1^2(-3b_1^2-4k^4\mu+k^4\lambda)}{4k^3}, g_2=0 \quad (3.13)$$

Where b_1,k are arbitrary constants.

Substituting (3.13) into (3.11) and (3.12), yields:

$$u(\xi)=\frac{2}{3}k^2(\tfrac{G'}{G})^2+\frac{2}{3}k^2\lambda(\tfrac{G'}{G})^1+\frac{1}{6}\frac{3b_1^2+4k^4\mu}{k^2}$$
(3.14)

$$v(\xi)=b_1(\tfrac{G'}{G})^1+\frac{1}{2}b_1\lambda$$
(3.15)

where $\xi=kx-\frac{3b_1^2}{2k}t$.

Substituting the general solutions of (3.10) into (3.14) and(3.15), we have:

When $\lambda^2-4\mu>0$

$$u_1(\xi)=-\frac{k^2\lambda^2}{6}+\frac{k^2}{6}(\lambda^2-4\mu).$$

$$(\frac{C_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi})^2$$

$$+\frac{1}{6}\frac{3b_1^2+4k^4\mu}{k^2}$$

$$v_1(\xi)=-\frac{1}{2}b_1\lambda+\frac{b_1\sqrt{\lambda^2-4\mu}}{2}.$$

$$\left(\frac{C_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\right)^2$$

$$+\frac{1}{2}b_1\lambda$$

where $\xi=kx-\frac{3b_1^2}{2k}t$, b_1,k are arbitrary constants.

When $\lambda^2-4\mu<0$

$$u_2(\xi)=-\frac{k^2\lambda^2}{6}+\frac{k^2}{6}(4\mu-\lambda^2).$$

$$\left(\frac{C_1\sinh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\cosh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\cosh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\sinh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}\right)^2$$

$$+\frac{1}{6}\frac{3b_1^2+4k^4\mu}{k^2}$$

$$v_2(\xi)=-\frac{1}{2}b_1\lambda+\frac{b_1\sqrt{4\mu-\lambda^2}}{2}.$$

$$\left(\frac{C_1\sinh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\cosh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\cosh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\sinh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}\right)^2$$

$$+\frac{1}{2}b_1\lambda$$

where $\xi=kx-\frac{3b_1^2}{2k}t$, b_1,k are arbitrary constants.

When $\lambda^2-4\mu=0$

$$u_3(\xi)=-\frac{k^2\lambda^2}{6}+\frac{k^2C_2^2}{3(C_1+C_2\xi)^2}+\frac{1}{6}\frac{3b_1^2+4k^4\mu}{k^2}$$

$$v_3(\xi)=\frac{b_1(2C_2-C_1\lambda-C_2\lambda\xi)}{2(C_1+C_2\xi)}+\frac{1}{2}b_1\lambda$$

where $\xi=kx-\frac{3b_1^2}{2k}t$, b_1,k are arbitrary constants.

IV. CONCLUSION

The main points of the (G'/G)-expansion method are thatassuming the solution of the ODE reduced by using thetraveling wave variable as well as integrating can be expre-ssedby an m-th degree polynomial in (G'/G), where $G=G(\xi)$ is the general solutions of a second order LODE.The positive integer m is determined by the homogeneousbalance between the highest order derivatives and nonlinearterms appearing in the reduced ODE, and the coefficientsof the polynomial can be obtained by solving a set ofsimu-ltaneous algebraic equations resulted from the processof using the method. Furthermore the method can also beused to many other nonlinear equations.

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