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Critical Design Parameters of Classical Rectangularssss Plate under Uniformly Distributed Lateral Load

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This study investigated the critical design parameters of classical rectangular SSSS plate under uniformly distributed lateral load. The study used third order total potential energy functional for isotropic rectangular thin plate with small deflection, external work was substituted into the third order total potential energy functional and the general equation of a classical rectangular plates under pure bending was obtained. The plate general equation was minimized with respect to deflection to obtain the equilibrium of forces governing equation of thin rectangular plate. The resistant forces were solved with Split-deflection approach and the solution gave the general polynomial deflection equation. Satisfying the boundary conditions of SSSS plate with respect to the general polynomial deflection equation gave the SSSS plate deflection equation. The general polynomial deflection equation was simplified and substituted into the plate governing equation to obtain the amplitude of deflection function, close integral was performed on the shape function for SSSS boundary conditions with respect to the general stiffness equation, which gave the peculiar stiffness and non-dimensional deflection coefficients of SSSS plate. Limit state conditions, such as ultimate limit state of stress $(\overline{\mathrm{U}} \leq \mathbb{U}_{\mathbf{0}})$and serviceability limit state of deflection $(W_{max} < W_a)$ were satisfied and the critical design parameters for thickness (t_c) and lateral imposed load (q_{ic})were obtained. Numerical examples were performed with the critical design equations and results were presented for critical design thicknesses (t_c) suitable to withstand a given set of loads and critical design imposed loads (q_{ic})a given thickness can withstand for SSSS plate.

Keywords: Pure Bending, Critical Thickness,Critical Impose Load,Limit State.

Notations: K: Stiffness of the material, σ: Stress, σ_x: x axis stress, σ_y: y axis stress, σ_z: z axis stress, τ_{yx}: y – x planer stress (shear stress in y – x plane), ε: Strain, ε_x: x axis strain, ε_y: y axis strain, ε_z: z axis strain, γ_{xy}: y – x planer strain (shear strain in y – x plane), L: Length of the material, E: Young modulus of elasticity, V: Work, U: Internal (strain) energy, D: Flexural rigidity of the plate, Π: Total potential energy of the plate, τ: Shear stress of the plate, ɣ: Shear strain of the plate, k_q: Load stiffness, k_x: Material stiffness on x plane, k_{xy}: Material stiffness on x – y plane, k_y: Material stiffness on y plane, x: Primary axis of the plate. That is the shorter of the two axes of the major plane of the plate, y: Secondary axis of the plate. That is the longer of the two axes of the major plane of the plate, z: Tertiary axis of the plate. That is the shortest of the three axes of the plate a: Length of the primary dimension of the plate, b: Length of the secondary dimension of the plate, t: Thickness of the plate or the length of the tertiary, w^i_x: The first derivative of the deflection in the x-axis, w^{ii}_x: The second derivative of the deflection in the x-axis, A: Amplitude of the deflection function (Coefficient of deflection), D: flexural rigidity of plate, µ: Poisson's ratio of plate material, R: Non dimension axis (quantity) parallel to x axis, A: Amplitude of the equation (Coefficient of deflection), h: Shape function, Q: Non dimension axis (quantity) parallel to y axis, W_{max} : Maximum deflection, W_a: Allowable deflection, q: Applied Load, φ: Unit weight of material, $\overline{\mathrm{U}}$: Total strain energy per volume, $\mathbb{U}_0$: Allowable total strain energy per volume, K_c: Maximum deflection Coefficient, SLS: Serviceability Limit State, ULS: Ultimate Limit

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State, SSSS:Four edges of the plate are simple supported, h_{max} : Deflected shape function at the center of the plate, q_{icD} : Critical Imposed load for deflection limit state pure bending analysis of rectangular classical plate, t_{cD} : Critical thickness deflection for limit state pure bending analysis of rectangular classical plate, q_{icE} : Critical Imposed load for elasticlimit state pure bending analysis of classical rectangular plate, t_{cE} : Critical thickness for elasticlimit state pure bending analysis of classical rectangular plate, q_{ic} : Critical Imposed load parameter, t_c: Critical thickness parameter.

I. INTRODUCTION

A plate is a structural component limited by two parallel planes called faces, and a cylindrical surface, called an edge. The division between the plane appearances is referred to as the thickness (t) of the plate, which it is common to isolate the thickness into equivalent parts by a plane parallel to its faces. This plane is known as the center plane (or basically, the mid-plane), where a and b are principal measurements, and t is the thickness (Yamaguchi, 1999). Plate is one of the continuum structure generally used in buildings, bridges, automobiles, hydraulic structures, pavements, containers, airplanes, missiles, ships, instruments, machine parts, table tops, street manhole covers, side panel, roof deck, tank bottom and so forth. As indicated by the definition applied to thin plate, the proportion of the thickness (t) to the smaller span length (a) should be less than 1/20 (Mansfield, 2005). We shall consider only small deflections of thin plates, which is a consistent magnitude of deformation found in plate structure. It is expected, except if generally indicated, that plate materials are homogeneous and isotropic. A homogenous material presents identical properties all through and when the material is the same in all directions, the material is called isotropic (Ventsel and Krauthammer, 2001). The maximum deflection of a laterally loaded plate has been obtained using the split deflection method (Ibearugbulem et al. 2016), the maximum deflection was used to satisfy SSSS plate boundary conditions so as to obtain the peculiar deflection equation of SSSS classical rectangular plate. With the non-dimensional total potential energy functional of a classical rectangular thin plate subjected to lateral load, the amplitude of the deflection function of SSSS plate was formulated.Also, we move further in getting the stiffness of SSSS plate before the critical parameter was solved. Satisfying the SLS of deflection and ULS of elasticity, the critical design parameters for Lateral impose load and thickness was obtained, this equation was used in solving for the critical lateral load a specified plate thickness can withstand and also critical thickness for a specified lateral load. From the literature, it has been discovered that there is no exploration by past researchers on the determination of critical design parameters of classical rectangular plates under uniformly distributed lateral load, a reason why the results presented in this study represent a novelty element brought by this research, which will be an advantage for plate designs. With this research a solution to critical load which a known plate thickness can withstand and also the critical thickness of a plate that can withstand a specified loading, can be known under specified conditions of operation.

II. THEORETICAL BACKGROUND

The study used Kirchhoff's hypotheses on total strain energy, work energy principle, kinematics, stress deflection relationship and constitutive relationship to derive the third order total potential energy functional for isotropic rectangular thin plate with small deflection, external work was substituted into the third order total potential energy functional and the general equation of a classical rectangular plates under pure bending was obtained. The plate general equation was minimized with respect to deflection to obtain the equilibrium of forces governing equation of thin rectangular plates. The resistant forces were solved with Split-deflection approach and the solution gave the general polynomial deflection equation.

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III. METHODOLOGY

The method used in this work is as presented below.

3.1 Deflection Function for (SS) Edge Condition (Simply Supported Edge)

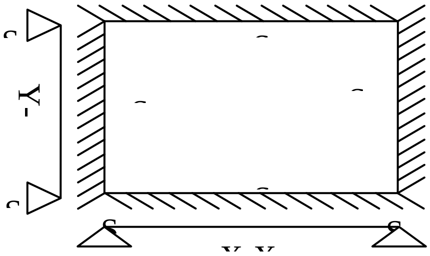

Figure 1: Representation of SSSS plate under lateral uniform load (q).

SS boundary conditions:

$$w_R(R=0)=0 \quad w_R''(R=0)=0 \qquad 1$$

$$w_R(R=1)=0 \quad w_R''(R=1)=0 \qquad 2$$

$$w_Q(Q=0)=0 \quad w_Q''(Q=0)=0 \qquad 3$$

$$w_Q(Q=1)=0 \quad w_Q''(Q=1)=0 \qquad 4$$

General orthogonal polynomial deflection equation of a plate is given by:

$$w=(a_0+a_1R+a_2R^2+a_3R^3+a_4R^4).(b_0+b_1Q+b_2Q^2+b_3Q^3+b_4Q^4) \qquad 5$$

Substituting the boundary conditions in Equations (1) to (4) into Equation (5) gives:

$$a_0=0,\ a_1=a_4,\ a_2=0,\ a_3=-2a_4, b_0=0,\ b_1=b_4,\ b_2=0,\ b_3=-2a_4 \qquad 6$$

Substituting Equations (6) into Equation (5) gives:

$$w_{SSSS}=a_4(R-2R^3+R^4).b_4(Q-2Q^3+Q^4) \qquad 7$$

Equation (7) is the Peculiar Deflection Function for SSSS Plate

When,

A= $a_4.b_4$ 8

h = $(R-2R^3+R^4).(Q-2Q^3+Q^4)$ 9

3.2Stiffness Coefficient for SSSS Classical Rectangular Plate.

From Equation (7), (8) and (9) gives:

$$\mathbf{w_{SSSS}=Ah} \qquad \mathbf{10}$$

When,

A= Amplitude of the deflection function (Coefficient of deflection)

h = Shape function

The non-dimensional third order total potential energy functional of a classical rectangular thin plate subjected to lateral load was formulated by Adewale in his master's degree thesis and presented as in Equation (11)

$$\Pi=\frac{D}{2}\int_0^1\int_0^1\left(\frac{\partial^3 Ah}{\partial R^3}.\frac{\partial Ah}{\partial R}+\frac{2}{\propto^2}.\frac{\partial^3 Ah}{\partial R\,\partial Q^2}.\frac{\partial Ah}{\partial R}+\frac{1}{\propto^4}.\frac{\partial^3 Ah}{\partial Q^3}.\frac{\partial Ah}{\partial Q}\right)\partial_R\,\partial_Q-\int_0^1\int_0^1 qa^4 Ah\ \partial_R\,\partial_Q \qquad 11$$

Differentiating Equation (11) with respect to(A) and equating to zero gives maximum value of A

$$DA\int_0^1\int_0^1\left(\frac{\partial^3 h}{\partial R^3}.\frac{\partial h}{\partial R}+\frac{2}{\propto^2}.\frac{\partial^3 h}{\partial R\,\partial Q^2}.\frac{\partial h}{\partial R}+\frac{1}{\propto^4}.\frac{\partial^3 h}{\partial Q^3}.\frac{\partial h}{\partial Q}\right)\partial_R\,\partial_Q-qa^4\int_0^1\int_0^1 h\,\partial_R\,\partial_Q=0 \qquad 12$$

$$A=\frac{qa^4\left(\int_0^1\int_0^1 h\right)\partial_R\,\partial_Q}{D\int_0^1\int_0^1\left(\frac{\partial^3 h}{\partial R^3}.\frac{\partial h}{\partial R}+\frac{2}{\propto^2}.\frac{\partial^3 h}{\partial R\,\partial Q^2}.\frac{\partial h}{\partial R}+\frac{1}{\propto^4}.\frac{\partial^3 h}{\partial Q^3}.\frac{\partial h}{\partial Q}\right)\partial_R\,\partial_Q} \qquad 13$$

Equation (13) can be written as shown in Equation (14)

$$A=\frac{qa^4}{D}\left(\frac{k_q}{k_x+\frac{2}{\propto^2}k_{xy}+\frac{1}{\propto^4}k_y}\right) 14$$

Where,

$$k_q=\int_0^a\int_0^a h\ \partial_R\partial_Q\ (Load\ stiffness) \qquad 15$$

$$k_x=\int_0^a\int_0^a\frac{\partial^3 h}{\partial R^3}.\frac{\partial h}{\partial R}\partial_R\partial_Q(Material\ stiffness\ in\ X\ direction)16$$

$$k_{xy}=\int_0^a\int_0^a\frac{\partial^3 h}{\partial R\partial Q^2}.\frac{\partial h}{\partial R}\partial_R\partial_Q\ (Material\ stiffness\ in\ x-y\ direction) \qquad 17$$

$$k_y=\int_0^a\int_0^a\frac{\partial^3 h}{\partial Q^3}.\frac{\partial h}{\partial Q}\partial_R\partial_Q(Material\ stiffness\ in\ y\ direction)18$$

Equation (14) can be written as shown in Equation (19)

$$A=\frac{qa^4}{D}.K \qquad 19$$

Where K is the total stiffness given by:

$$K=\frac{k_q}{k_x+\frac{2}{\propto^2}k_{xy}+\frac{1}{\propto^4}k_y} \qquad 20$$

3.3 Determination of the Coefficient of Deflection (A)

The stiffness coefficient of SSSS classical rectangular plate from Equation (15) to Equation (18) can be solved by definite integration of the shape functions deflection (h)in Equation (9) from 0 to 1. K_q, K_x, K_{xy} and $K_y$are respectively 0.04, 0.2361904762, 0.235918 and 0.2361904762.

Substituting the values of the coefficients K_q, K_x, K_{xy} and $K_y$into Equation (14) gives:

$$A=\frac{qa^4}{D}\left(\frac{0.04}{0.2361904762+\frac{2}{\propto^2}0.235918+\frac{1}{\propto^4}0.2361904762}\right) \qquad 21$$

3.4Determination of Critical Design Parameters of Classical Rectangular Plates under Uniformly Distributed Lateral Load

3.4.1 Serviceability Limit State of Deflection Pure Bending Analysis of Classical Rectangular Plate

From Deflection Limit State which states that the maximum deflection is less than allowable deflection and this can be mathematically written as:

$$W_{max}<W_a \qquad 22$$

When,

$W_{max}=Maximum\ deflection$

$W_a=Allowable\ deflection$

Substitution of Equation (19) into Equation (9) gives:

$$w=\frac{qa^4}{D}.K.h \qquad 23$$

For maximum deflection Equation (23) can be expressed as:

$$W_{max}=\frac{qa^4}{D}.K.h_{max} \qquad 24$$

When,

h_{max} = The point of maximum stress of a lateral loaded classical plate and this occurs at the center of the plate.

Substituting Equation (24) into the deflection Limit state condition in Equation (22) gives:

$$\frac{qa^4}{D}.K.h_{max} < W_a \quad 25$$

3.4.1.1 Determination of Critical Imposed Load for Deflection Limit State Pure Bending Analysis of Classical Rectangular Plate

Solving for the critical lateral loading from Equation (25) gives:

$$q < \frac{W_a \cdot D}{K.h_{max} \cdot a^4} \quad 26$$

$$D = Flexural\ ridgity\ of\ the\ plate = \frac{t^3 \cdot E}{12\,(1-\mu^2)} \quad 27$$

Substituting Equation (27) into Equation (26) gives:

$$q < \frac{W_a \cdot t^3 \cdot E}{K.h_{max} \cdot a^4 \cdot 12\,(1-\mu^2)} \quad 28$$

From Euro code 1 EN 1991-1-1 andMccormac et al, (2012, 2014),

$$If\ q = q_s + q_i = q_i + (\varphi \cdot t) \quad 29$$

When,

$q = Applied\ load;\ q_i = Imposed\ Load;\ t = Thickness\ of\ the\ plate;\ q_s = Dead\ load$

$\varphi = Unit\ weight\ of\ material$

Substituting Equation (29) into Equation (28) gives:

$$q_i < \frac{W_a \cdot t^3 \cdot E}{K.h_{max} \cdot a^4 \cdot 12\,(1-\mu^2)} - (\varphi \cdot t) \quad 30$$

Let

$$\emptyset_1 = \frac{1}{12.K.h_{max}} \quad 31$$

Rewriting Equation (31) gives:

$$q_{icD} < \emptyset_1 \frac{W_a \cdot t^3 \cdot E}{a^4 \cdot (1-\mu^2)} - (\varphi \cdot t) \quad 32$$

3.4.1.2 Determination of Critical Thickness for Deflection Limit State Pure Bending Analysis of Classical Rectangular Plate

From Equation (28) the critical thickness (t_{cD}) equation can be derived when the load is known and it's given as:

$$\frac{q.K.h_{max} \cdot a^4 \cdot 12\,(1-\mu^2)}{W_a.E} < t^3 \quad 33$$

Solving for the critical thickness from Equation (33) gives:

$$t > \sqrt[3]{\frac{q.K.h_{max} \cdot a^4 \cdot 12\,(1-\mu^2)}{W_a.E}} \quad 34$$

Let

$$\emptyset_2 = \sqrt[3]{12.K.h_{max}} \quad 35$$

Rewriting Equation (35) gives:

$$t_{cD} > \emptyset_2 \left(\frac{q.a^4(1-\mu^2)}{W_a.E}\right)^{\frac{1}{3}} \quad 36$$

Equation (32) is the critical imposed load equation(q_{icD}), a classical rectangular plate thickness can withstand at specified thickness and deflection.

Equation (36) is the critical thickness equation (t_{cD}) of the rectangular plate, such that it can carry a specified lateral load at a specified deflection.

3.4.2 Ultimate Limit State of Stress forPure Bending Analysis of Classical Rectangular Plate

From Elasticity theory according to Ibearugbulem (2017), the strain energy limit state is stated as:

$$(\bar{U} \leq \mathbb{U}_0) \quad 37$$

Where,

$\bar{U} = Total\ strain\ energy\ per\ volume;\ \mathbb{U}_0 = Allowable\ total\ strain\ energy\ per\ volume$

This is done in line with the work of Ibearugbulem (2017), allowable total strain energy is:

$$\frac{1}{2E}\left(\sigma_x{}^2+\sigma_y{}^2+2\tau_{xy}{}^2+2\mu\left(\tau_{yx}{}^2-\sigma_y\sigma_x\right)\right) \leq \frac{f_y{}^2}{2E} \quad 38$$

Simplifying Equation (38) gives:

$$\left(\sigma_x{}^2+\sigma_y{}^2+2\tau_{xy}{}^2+2\mu\left(\tau_{yx}{}^2-\sigma_y\sigma_x\right)\right) \leq f_y{}^2 \quad 39$$

Let the ratios relating σ_x , σ_y and τ_{xy} be given as:

$$\sigma_y = n_1\sigma_x \quad 40$$

$$\tau_{xy} = n_2\sigma_x \quad 41$$

Substituting Equation (40) and Equation (41) into Equation (39) gives:

$$\sigma_x{}^2+(n_1\sigma_x)^2+2(n_2\sigma_x)^2+2\mu((n_2\sigma_x)^2-(n_1\sigma_x)\sigma_x) \leq f_y{}^2 \quad 42$$

Rearranging Equation (42) Taking Square root gives:

$$\sigma_x \leq \frac{f_y}{\sqrt{(1+(n_1)^2+2n_2{}^2+2\mu n_2{}^2-2\mu n_1)}} \quad 43$$

Equation (43) can be called critical stress.

From Equation (40) and Equation (41) can rewritten as:

$$n_1 = \frac{\sigma_y}{\sigma_x} \quad 44$$

$$n_2 = \frac{\tau_{xy}}{\sigma_x} \quad 45$$

The solution of stresses acting on a classical rectangular thin plate is given as:

$$\sigma_x = \frac{-zE}{1-\mu^2}\left(\mu\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial x^2}\right) 46$$

$$\sigma_y = \frac{-zE}{1-\mu^2}\left(\mu\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right) 47$$

$$\tau_{yx} = \frac{-zE(1-\mu)}{(1-\mu^2)}.\frac{\partial^2 w}{\partial y\partial x} 48$$

Substituting w = Ah, the dimensionless coordinates x = aR, y = bQ, aspect ratio ∝ = b/a into the stress solution in Equation (46) to Equation (48)

$$\sigma_y = \frac{-zEA}{1-\mu^2}\left(\mu\frac{\partial^2 h}{\partial R^2}+\frac{\partial^2 h}{\propto^2\partial Q^2}\right) \quad 49$$

$$\sigma_x = \frac{-zEA}{1-\mu^2}\left(\mu\frac{\partial^2 h}{\propto^2\partial Q^2}+\frac{\partial^2 h}{\partial R^2}\right) \quad 50$$

$$\tau_{yx} = \frac{-zEA(1-\mu)}{(1-\mu^2)}.\frac{\partial^2 h}{\propto\partial R\partial Q} \quad 51$$

Substituting Equation 49, 50 and 51 into Equation 44 and Equation 45 gives:

$$n_1 = \frac{\left(\mu\frac{\partial^2 h}{\partial R^2}+\frac{\partial^2 h}{\propto^2\partial Q^2}\right)}{\left(\mu\frac{\partial^2 h}{\propto^2\partial Q^2}+\frac{\partial^2 h}{\partial R^2}\right)} \quad 52$$

$$n_2 = \frac{(1-\mu).\frac{\partial^2 h}{\propto\partial R\partial Q}}{\left(\mu\frac{\partial^2 h}{\propto^2\partial Q^2}+\left(\frac{\partial^2 h}{\partial R^2}\right)\right)} \quad 53$$

Let the second derivative of the shape function stresses be denoted as:

$$\Phi_{y_1} = \frac{\partial^2 h}{\partial Q^2} \quad 54$$

$$\Phi_{x_1} = \frac{\partial^2 h}{\partial R^2} \quad 55$$

$$\Phi_{xy_1} = \frac{\partial^2 h}{\partial R \partial Q} \quad 56$$

Substituting Equation (54), (55), and Equation (56) into Equation (52) and Equation (53) gives:

$$n_1 = \frac{\left(\mu \Phi_{x_1} + \frac{1}{\alpha^2}\Phi_{y_1}\right)}{\left(\mu \frac{1}{\alpha^2}\Phi_{y_1} + \Phi_{x_1}\right)} \quad 57$$

$$n_2 = \frac{(1-\mu).\frac{1}{\alpha}\Phi_{xy_1}}{\left(\mu \frac{1}{\alpha^2}\Phi_{y_1} + \Phi_{x_1}\right)} \quad 58$$

Substituting Equation (57) and Equation (58) into Equation (43), so the critical stress can be rewritten as:

$$\sigma_x \le \frac{f_y}{\sqrt{1 + \left(\frac{\mu\Phi_{x_1} + \frac{1}{\alpha^2}\Phi_{y_1}}{\mu\frac{1}{\alpha^2}\Phi_{y_1} + \Phi_{x_1}}\right)^2 + 2\left(\frac{(1-\mu).\frac{1}{\alpha}\Phi_{xy_1}}{\mu\frac{1}{\alpha^2}\Phi_{y_1} + \Phi_{x_1}}\right)^2 + 2\mu\left(\frac{(1-\mu).\frac{1}{\alpha}\Phi_{xy_1}}{\mu\frac{1}{\alpha^2}\Phi_{y_1} + \Phi_{x_1}}\right)^2 - 2\mu\left(\frac{\mu\Phi_{x_1} + \frac{1}{\alpha^2}\Phi_{y_1}}{\left(\mu\frac{1}{\alpha^2}\Phi_{y_1} + \Phi_{x_1}\right)}\right)}} \quad 59$$

Let

$$n = \sqrt{1 + \left(\frac{\mu\Phi_{x_1} + \frac{1}{\alpha^2}\Phi_{y_1}}{\mu\frac{1}{\alpha^2}\Phi_{y_1} + \Phi_{x_1}}\right)^2 + 2\left(\frac{(1-\mu).\frac{1}{\alpha}\Phi_{xy_1}}{\mu\frac{1}{\alpha^2}\Phi_{y_1} + \Phi_{x_1}}\right)^2 + 2\mu\left(\frac{(1-\mu).\frac{1}{\alpha}\Phi_{xy_1}}{\mu\frac{1}{\alpha^2}\Phi_{y_1} + \Phi_{x_1}}\right)^2 - 2\mu\left(\frac{\mu\Phi_{x_1} + \frac{1}{\alpha^2}\Phi_{y_1}}{\left(\mu\frac{1}{\alpha^2}\Phi_{y_1} + \Phi_{x_1}\right)}\right)} \quad 60$$

Substituting Equation (60) into Equation (59) gives:

$$\sigma_x \le \frac{f_y}{n} \quad 61$$

Substituting Equation (54) and Equation (55) into Equation (50) gives:

$$\sigma_x = \frac{-zEA}{1-\mu^2}\left(\mu\frac{\Phi_{y_1}}{\alpha^2} + \Phi_{x_1}\right) \quad 62$$

Substituting Equation (62) into Equation (61) gives:

$$\frac{-zEA}{1-\mu^2}\left(\mu\frac{\Phi_{y_1}}{\alpha^2} + \Phi_{x_1}\right) \le \frac{f_y}{n} \quad 63$$

Substituting Equation (19) and mid plane Z=t/2into Equation (63) gives:

$$\left(\frac{-t\,.E\,.q\,.a^4.K\,.\left(\mu\frac{1}{\alpha^2}\Phi_{y_1} + \Phi_{x_1}\right)}{2\,.D.(1-\mu^2)}\right) \le \frac{f_y}{n} \quad 64$$

Substituting Equation (27) and Equation (29) into Equation (64) gives:

$$q_i + (\varphi\,.t) \le \frac{f_y\,.t^2\,.(1-\mu^2)}{-6.K\,.\,n\,.(1-\mu^2).a^4\,.\left(\mu\frac{1}{\alpha^2}\Phi_{y_1} + \Phi_{x_1}\right)} \quad 65$$

Let

$$\emptyset_3 = \frac{1}{-6.K} \quad 66$$

Substituting Equation (66) into Equation (65) to get the critical imposed lateral load and this gives:

$$q_{icE} \leq \frac{\emptyset_3 . f_y \ . t^2}{n \ . a^4 . \left(\mu \frac{1}{\propto^2} \Phi_{y_1} + \Phi_{x_1}\right)} - (\varphi \ . t) 67$$

3.4.2.3 Determination of Critical Thickness for Elastic Limit State Pure Bending Analysis of Classical Rectangular Plate

From Equation (64) the critical thickness (t_{cE}) can be derived when the load is known and it's given as:

$$t_c{}^2 \geq \frac{-1 \ . q. \ n \ . a^4 . \left(\mu \frac{1}{\propto^2} \Phi_{y_1} + \Phi_{x_1}\right) . 12 \ (1-\mu^2) . k}{f_y \ . 2 \ . (1-\mu^2)} 68$$

Let,

$$\emptyset_4 = (6. k)^{\frac{1}{2}} \quad 69$$

Substituting Equation (69) into Equation (68) to get the critical thickness and this gives:

$$t_{cE} \geq \emptyset_4 \left(\frac{-1. q. \ n \ . a^4 . \left(\mu \frac{1}{\propto^2} \Phi_{y_1} + \Phi_{x_1}\right)}{f_y} \right)^{\frac{1}{2}} \quad 70$$

Equation (67) is the critical imposed load equation(q_{icE}), a classical rectangular plate thickness can withstand at specified thickness and material strength.

Equation (70) is the critical thickness equation (t_{cE}) of classical rectangular plate such that it can carry a specified lateral load at a specified material strength.

Equations of critical design parameters of classical rectangular plates under uniformly distributed lateral load are:

$$q_{icD} < \emptyset_1 \frac{W_a \ . \ t^3 \ . E}{a^4 . (1-\mu^2)} - \varphi \ . t \quad 71$$

$$t_{cD} > \emptyset_2 \left(\frac{q. a^4 (1-\mu^2)}{W_a . E} \right)^{\frac{1}{3}} \quad 72$$

$$q_{icE} \leq \frac{\emptyset_3 . f_y \ . t^2}{n \ . a^4 . \left(\mu \frac{1}{\propto^2} \Phi_{y_1} + \Phi_{x_1}\right)} - (\varphi \ . t) \quad 73$$

$$t_{cE} \geq \emptyset_4 \left(\frac{-1. q. \ n \ . a^4 . \left(\mu \frac{1}{\propto^2} \Phi_{y_1} + \Phi_{x_1}\right)}{f_y} \right)^{\frac{1}{2}} \quad 74$$

3.4.3 Determination of Maximum Stress Coefficient for SSSS Boundary Conditions.

The point of maximum stress for a classical rectangular plate under uniformly distributed lateral load occurs at the center of the plate and this can be mathematically represented as:

h_{max} occurs at $(R = Q = 0.5)$.

3.4.3.1 Maximum Stress Coefficient for SSSS Classical Rectangular Plate

From Equation (9), the shape function of SSSS plate is given as

$$h = \left((R - 2R^3 + R^4). (Q - 2Q^3 + Q^4)\right)$$

$$h_{max} = \left((0.5 - 2(0.5)^3 + 0.5^4). \left((0.5 - 2(0.5)^3 + 0.5^4)\right)\right) = 0.0977 \quad 75$$

Substituting Equation 9 into Equation 55 gives:

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$$\Phi_{x_1} = \frac{\partial^2 h}{\partial R^2} = 12(0.5^2 - 0.5)(0.5 - [2 * 0.5^3] + 0.5^4) = -0.9375 \quad 76$$

Substituting Equation 9 into Equation 56 gives:

$$\Phi_{xy_1} = \frac{\partial^2 h}{\partial R \partial Q} = (1 - [6 * 0.5^2] + [4 * 0.5^3])(1 - [6 * 0.5^2] + [4 * 0.5^3]) = 0.0000 \quad 77$$

Substituting equation 9 into Equation 54 gives:

$$\Phi_{y_1} = \frac{\partial^2 h}{\partial Q^2} = 12(0.5^2 - 0.5)(0.5 - [2 * 0.5^3] + 0.5^4) = -0.9375 \quad 78$$

Numerical Examples

Numerical examples were performed using the critical design (limit) parameters listed in Equation (71), Equation (72),Equation (73) and Equation (74), and the parameter used for this example are as follows in Table 1:

Table 1: Parameters for Numerical Examples

SYMBOLS	VALUES
E	$207 \times 10^9\ N/m^2$
M	0.3
Φ	$77\ kN/m^3$
A	$1m$
f_y	250N/mm^2, 415N/mm^2,
W_a	5mm, 10mm, 15mm
T	5mm, 10mm, 15mm, 20mm
$\alpha = \frac{b}{a}$	1, 1.5, 2
q	50kN, 100kN, 150kN, 200kN

IV. RESULTS AND DISCUSSION

For critical lateral imposed load numerical studies, plate thicknesses of 5mm, 10mm, 15mm, 20mm were considered. The specified deflections, material strength, physical and geometric properties above and Table 2 were substituted into the critical lateral imposed load equation for serviceability limit state of deflection (q_{icD}) in Equation (71) and also the critical lateral imposed load equation for ultimate limit state of stress (q_{icE}) in Equation (73). Results of this substitutions with respect to the considered aspect ratios in the numerical examples parameters gave the critical lateral imposed load from Table 3 to Table 4 and Table 5, choosing the lesser load between the deflection and the stress loads of a specified plate thickness, aspect ratio and boundary condition. This load is said to be the critical lateral imposed load the plate thickness can withstand without failure and also satisfying the design limit state conditions.

For critical thickness numerical studies, lateral loads, 50kN, 100kN, 150kN and 200kN were considered. The specified deflections, material strength, physical and geometric properties above and Table 2 were substituted into the critical thickness equation for serviceability limit state of deflection (t_{cD}) in Equation (72) and also the critical thickness equation for ultimate limit state of stress (t_{cE}) in Equation (74). Results of this substitutions with respect to the considered aspect ratios in the numerical examples parameters were tabulated on Table 6, Table 7 and Table 8 and the critical thicknesses was selected choosing the larger thickness between the deflection and the stress thicknesses of a specified loads intensity, aspect ratio and boundary condition. This thickness is said to be the critical classical plate thickness that can withstand the specified lateral load intensity without failure and also satisfying the design limit state conditions.

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Table 2

$\alpha = \frac{b}{a}$	$Stiffness$ (K	$\emptyset_1$	$\emptyset_2$	$\emptyset_3$	$\emptyset_4$	Value on **n** with Poisson's ratio, $\mu = 0.3$
1.0	0.042363	20.13427	0.367582	-3.9342	0.50416	2.6
1.5	0.08121	10.50303	0.456627	-2.0523	0.69804	1.8256
2.0	0.108427	7.866585	0.502811	-1.5371	0.80658	1.5687

4.1.5.3 Critical Lateral Imposed Loads for Classical Rectangular Plates

Table 3: Critical lateral imposed loads for SSSS classical rectangular plate on aspect ratio of 1, under specified allowable deflections, thicknesses and material strengths.

Critical lateral imposed loads (q_{ic})						
t(mm)	$Aspect\ ration\ \propto = \frac{b}{a} = 1$					
	W_a = 5mm	f_y =250N/mm^2	q_{ic} (kN)	W_a = 5mm	f_y =415N/mm^2	q_{ic} (kN)
5	2.4775	7.37484	2.4775	2.4775	12.4963	2.47
10	22.13	30.2694	22.13	22.13	50.7553	22.1
15	76.1324	68.6835	68.6835	76.1324	114.777	76.1
20	181.66	122.617	122.617	181.66	204.561	181.
t(mm)	W_a = 10mm	f_y =250N/mm^2	q_{ic} (kN)	W_a = 10mm	f_y =415N/mm^2	q_{ic} (kN)
5	5.33999	7.37484	5.33999	5.33999	12.4963	5.33
10	45.0299	30.2694	30.2694	45.0299	50.7553	45.0
15	153.42	68.6835	68.6835	153.42	114.777	114.
20	364.86	122.617	122.617	364.86	204.561	204.
t(mm)	W_a = 15mm	f_y =250N/mm^2	q_{ic} (kN)	W_a = 15mm	f_y =415N/mm^2	q_{ic} (kN)
5	8.20249	7.37484	7.37484	8.20249	12.4963	8.20
10	67.9299	30.2694	30.2694	67.9299	50.7553	50.7
15	230.707	68.6835	68.6835	230.707	114.777	114.
20	548.059	122.617	122.617	548.059	204.561	204.

Table 4: Critical lateral imposed loads for SSSS classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, thicknesses and material strengths.

Critical lateral imposed loads (q_{ic})						
t(mm)	$Aspect\ ration\ \propto = \frac{b}{a} = 1.5$					
	W_a = 5mm	f_y =250N/mm^2	q_{ic} (kN)	W_a = 5mm	f_y =415N/mm^2	q_{ic} (kN)
5	1.10822	6.22784	1.10822	1.10822	10.5923	1.10822
10	11.1758	25.6814	11.1758	11.1758	43.1393	11.1758

15	39.1619	58.3606	39.1619	39.1619	97.6408	39.1619
20	94.0261	104.265	94.0261	94.0261	174.097	94.0261
t(mm)	W_a = 10mm	f_y=250N/mm^2	q_{ic} (kN)	W_a = 10mm	f_y=415N/mm^2	q_{ic} (kN)
5	2.60144	6.22784	2.60144	2.60144	10.5923	2.60144
10	23.1215	25.6814	23.1215	23.1215	43.1393	23.1215
15	79.4789	58.3606	58.3606	79.4789	97.6408	79.4789
20	189.592	104.265	104.265	189.592	174.097	174.097
t(mm)	W_a = 15mm	f_y=250N/mm^2	q_{ic} (kN)	W_a = 15mm	f_y=415N/mm^2	q_{ic} (kN)
5	4.09466	6.22784	4.09466	4.09466	10.5923	4.09466
10	35.0673	25.6814	25.6814	35.0673	43.1393	35.0673
15	119.796	58.3606	58.3606	119.796	97.6408	97.6408
20	285.158	104.265	104.265	285.158	174.097	174.097

Table 5: Critical lateral imposed loads for SSSS classical rectangular plate on aspect ratio of 2, under specified allowable deflections, thicknesses and material strengths.

Critical lateral imposed loads (q_{ic})						
t(mm)	$Aspect\ ration\ \propto = \frac{b}{a} = 2$					
	W_a = 5mm	f_y=250N/mm^2	q_{ic} (kN)	W_a = 5mm	f_y=415N/mm^2	q_{ic} (kN)
5	0.7334	5.69159	0.7334	0.7334	9.70215	0.7334
10	8.17716	23.5364	8.17716	8.17716	39.5786	8.17716
15	29.0417	53.5343	29.0417	29.0417	89.6293	29.0417
20	70.0373	95.6855	70.0373	70.0373	159.854	70.0373
t(mm)	W_a = 10mm	f_y=250N/mm^2	q_{ic} (kN)	W_a = 10mm	f_y=415N/mm^2	q_{ic} (kN)
5	1.85179	5.69159	1.85179	1.85179	9.70215	1.85179
10	17.1243	23.5364	17.1243	17.1243	39.5786	17.1243
15	59.2383	53.5343	53.5343	59.2383	89.6293	59.2383
20	141.615	95.6855	95.6855	141.615	159.854	141.615
t(mm)	W_a = 15mm	f_y=250N/mm^2	q_{ic} (kN)	W_a = 15mm	f_y=415N/mm^2	q_{ic} (kN)
5	2.97019	5.69159	2.97019	2.97019	9.70215	2.97019
10	26.0715	23.5364	23.5364	26.0715	39.5786	26.0715
15	89.435	53.5343	53.5343	89.435	89.6293	89.435
20	213.192	95.6855	95.6855	213.192	159.854	159.854

4.1.5.4 Critical Thicknesses for Classical Rectangular Plates

Table 6: Critical thicknesses for SSSS classical rectangular plate on aspect ratio of 1, under specified allowable deflections, lateral imposed loads and material strengths.

Critical thicknesses (t_c)

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q(kN)	Aspect ration $\propto = \frac{b}{a} = 1$					
	W_a = 5mm	f_y=250N/mm^2	t_c(m)	W_a = 5mm	f_y=415N/mm^2	t_c(m)
50	0.01297	0.01269	0.01297	0.01297	0.00985	0.01297
100	0.01635	0.01795	0.01795	0.01635	0.01393	0.01635
150	0.01871	0.02198	0.02198	0.01871	0.01706	0.01871
200	0.02059	0.02538	0.02538	0.02059	0.0197	0.02059
q(kN)	W_a = 10mm	f_y=250N/mm^2	t_c(m)	W_a = 10mm	f_y=415N/mm^2	t_c(m)
50	0.0103	0.01269	0.01269	0.0103	0.00985	0.0103
100	0.01297	0.01795	0.01795	0.01297	0.01393	0.01393
150	0.01485	0.02198	0.02198	0.01485	0.01706	0.01706
200	0.01635	0.02538	0.02538	0.01635	0.0197	0.0197
q(kN)	W_a = 15mm	f_y=250N/mm^2	t_c(m)	W_a = 15mm	f_y=415N/mm^2	t_c(m)
50	0.009	0.01269	0.01269	0.009	0.00985	0.00985
100	0.01133	0.01795	0.01795	0.01133	0.01393	0.01393
150	0.01297	0.02198	0.02198	0.01297	0.01706	0.01706
200	0.01428	0.02538	0.02538	0.01428	0.0197	0.0197

Table 7: Critical thicknesses for SSSS classical rectangular plate on aspect ratio of 1.5, under specified allowable deflections, lateral imposed loads and material strengths.

Critical thicknesses (t_c)						
q(kN)	Aspect ration $\propto = \frac{b}{a} = 1.5$					
	W_a = 5mm	f_y=250N/mm^2	t_c(m)	W_a = 5mm	f_y=415N/mm^2	t_c(m)
50	0.01612	0.01375	0.01612	0.01612	0.01067	0.01612
100	0.02031	0.01944	0.02031	0.02031	0.01509	0.02031
150	0.02324	0.02381	0.02381	0.02324	0.01848	0.02324
200	0.02558	0.0275	0.0275	0.02558	0.02134	0.02558
q(kN)	W_a = 10mm	f_y=250N/mm^2	t_c(m)	W_a = 10mm	f_y=415N/mm^2	t_c(m)
50	0.01279	0.01375	0.01375	0.01279	0.01067	0.01279
100	0.01612	0.01944	0.01944	0.01612	0.01509	0.01612
150	0.01845	0.02381	0.02381	0.01845	0.01848	0.01848
200	0.02031	0.0275	0.0275	0.02031	0.02134	0.02134
q(kN)	W_a = 15mm	f_y=250N/mm^2	t_c(m)	W_a = 15mm	f_y=415N/mm^2	t_c(m)
50	0.01117	0.01375	0.01375	0.01117	0.01067	0.01117
100	0.01408	0.01944	0.01944	0.01408	0.01509	0.01509
150	0.01612	0.02381	0.02381	0.01612	0.01848	0.01848

Table 8: Critical thicknesses for SSSS classical rectangular plate on aspect ratio of 2, under specified allowable deflections, lateral imposed loads and material strengths.

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Critical thicknesses (t_c)						
q(kN)	Aspect ration $\propto = \frac{b}{a} = 2$					
	W_a = 5mm	f_y=250N/mm^2	t_c(m)	W_a = 5mm	f_y=415N/mm^2	t_c(m)
50	0.01775	0.01434	0.01775	0.01775	0.01113	0.01775
100	0.02236	0.02028	0.02236	0.02236	0.01574	0.02236
150	0.02559	0.02484	0.02559	0.02559	0.01928	0.02559
200	0.02817	0.02869	0.02869	0.02817	0.02226	0.02817
q(kN)	W_a = 10mm	f_y=250N/mm^2	t_c(m)	W_a = 10mm	f_y=415N/mm^2	t_c(m)
50	0.01409	0.01434	0.01434	0.01409	0.01113	0.01409
100	0.01775	0.02028	0.02028	0.01775	0.01574	0.01775
150	0.02031	0.02484	0.02484	0.02031	0.01928	0.02031
200	0.02236	0.02869	0.02869	0.02236	0.02226	0.02236
q(kN)	W_a = 15mm	f_y=250N/mm^2	t_c(m)	W_a = 15mm	f_y=415N/mm^2	t_c(m)
50	0.0123	0.01434	0.01434	0.0123	0.01113	0.0123
100	0.0155	0.02028	0.02028	0.0155	0.01574	0.01574
150	0.01775	0.02484	0.02484	0.01775	0.01928	0.01928
200	0.01953	0.02869	0.02869	0.01953	0.02226	0.02226

CONCLUSIONS

Based on the research results obtained from this present study, the boundary conditions, aspect ratios, allowable deflections and material strength plays a significant effect on the critical lateral imposed load and critical thicknessof classical rectangular plate.

The critical design parameters tables q_{ic} and t_c herein are very reliable and can be used in the determination of suitable plate thickness from a specified lateral load and also the critical lateral imposed load a specified plate thickness can withstand, under specified condition of operations.

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