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Chaotheory: Studying the Behaviour of Dynamical Systems and Its High Sensitivity to Initial Conditions and Retrieving Of Coded Signal via Response System

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ABSTRACT
In this research work, synchronization of four dimensional hyperchaotic Lorenz system (Driver System) with Chen's system (Response system) are investigated based on backstepping technique, unlike the well known i = j, multi-switching of the indices was employed in the usual master-slave synchronization scheme. We provided varieties of non-identical indices, that is, i = j of the driver system. In this high dimension, more switching options for constructing the error space vector due to the large number of variables are available; and by implication providing variety of synchronization directions between the variables of the master and the slave systems, which could be used in securing electronic information and communication.

Keywords: Synchronization, multi-switching, hyperchaotic, dynamical system.

I. INTRODUCTION

Chaos theory is a branch of mathematics focused on the behavour of dynamical systems that is highly sensitive to initial conditions – a response popularly referred to as the butterfly effect. Dynamical system are deterministic, whose feature behaviour is fully determined by their initial conditions, with no random elements involved.

The dynamical system concept is a mathematical formalization for any fixed "rule" which describes the time dependence of a points position in its ambient space. This work is deterministic in nature, and as such and being a deterministic model has always produces the same output from a given starting condition or initial state.

It has been found or discovered that Non linear deterministic dynamical systems exhibits sensitive dependence on initial conditions. However, different methods have been employed to describe their existence in the fields of sciences, medicine and engineering Strogatz (2000) [1]. Let us note that amongst the attributes of nonlinear dynamical systems such as chaos, bifurcation, multi-stability, pattern formation and synchronization have been found very useful in many disciplines.

The study of behaviour of dynamical system cannot be studied in isolation without synchronization of chaotic and hyperchaotic systems, which has been referred to as a major break through [2] and one of the most important attributes of nonlinear dynamical systems, because of its potential applications in modelling brain activities, chemical reactions and more importantly in information processing and secure communication (coded signal).

More discoveries in the various types of synchronization came into the open due to increasing interest in the study of synchronization of chaotic systems. Such types of synchronization include complete synchronization [3], phase synchronization [4], lag synchronization [5], generalized synchronization [6], [7], measure synchronization [8] [9] and [10], projective synchronization [11], [12], and [13], anticipated synchronization [14], [15], reduced-order synchronization [16] and function projective [17].

Many outstanding methods of achieving synchronization between two or more non-linear systems have been proposed and well developed. These include, among others the Adaptive control [18], active control and robust synchronization [19], impulsive control [20], adaptive fuzzy feedback [21], sliding mode control [22] and back stepping technique [23].

Backstepping technique did show or exhibit outstanding performance in the synchronization of identical and non-identical chaotic systems, stabilization and tracking [24] and controlling of hyperchaotic systems [25], and useful in either the strict feedback or the non-strict feedback systems [26].

The temporal complexity and apparent randomness of chaotic systems is the most important characteristics of chaos [27]. So, the primary motivation or purpose of synchronization is that, one can hide certain electronic information to be transmitted in chaotic signal and retrieve by the technique of chaotic synchronization. Nevertheless, Meng et al [27] opined that absolute security of information and communication based on low dimensional chaotic [2] and hyperchaotic [28] systems, cannot be fully guaranteed. The reason being that, it can be reconstructed easily and separated from the secure information.

Therefore, concerted efforts were made to generate higher dimensional systems. In this direction, 4-dimensional systems have been studied, most of which are known to exhibit instability in two directions implying that the possibility for the existence of two positive Lyapunov exponent is ascertained. This paper certainly investigated 4-dimensional hyperchaotic system. More interestingly, there is the 5D hyperchaotic system coined by Yang and Chen (2013) [29].

In this work – multi-switching synchronization of chaotic system, we investigated the multi-switching of coupled systems with active controls. The synchronization is achieved with the slave-master scheme, and it is extended to investigate the synchronisation problems with different combinations of slave states with master systems. The different active controller is designed for the chosen slave system state to be synchronised with the target master system states. The multi-switching synchronisation of Lorenz system is considered. Illustration provided that combination synchronisation as a scheme would achieve for the multi-switching synchronisation to combination synchronisation such that the state variables of two or more driving systems synchronise with different variables of the response system, simultaneously.

II. SYSTEM DESCRIPTION

Based on the Lorenz system, a new chaotic system was reported by [30]which has attracted the attention of many researchers [31]–[34]. The Lorenz chaotic system is described by

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= cx - xz \\ \dot{z} &= xy - bz \end{aligned} \tag{1}$$

Where a,b and c are positive real constant. It is a 3D autonomous system with six terms including only two quadratic terms in a form very similar to the Lorenz, Chen and Lu systems in [35] – [38], but it has three very different fixed points: one saddle and two stable node-foci. By introducing a linear feedback controller to the second equation of the Lorenz-like system (1), the following new hyperchaotic system is obtained [39].

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= cx + y - xz - w \\ \dot{z} &= xy - bz \\ \dot{w} &= dyz \end{aligned} \tag{2}$$

The attractors of the system at the states x – y, x – z, x – w, y – z, y – w, z – x, z – y and z – w are as shown in the upper part of figure 1 which the time series of the combined state of the system is as shown in the lower part.

Decryption of the Hyperchaotic Chen's model

The hyperchaotic Chen system is given by

$$\begin{aligned} \dot{x} &= a(y-x) + w \\ \dot{y} &= dx - xz + cy \\ \dot{z} &= xy - bz \\ \dot{w} &= dy + ew \end{aligned} \tag{3}$$

Where a = 35, b = 3, c = 12, d = 7, e = 0.5 are parameters. The attractors of the system at the states x – y, x – z, x – w, y – z, y – w, z – x, z – y and z – w are as shown in the upper part while the time series is shown in the lower part of figure 2.

III. DEFINITION AND FORMULATION OF MULTISWITCHING SYNCHRONIZATION

Considering the following master-slave n dimensional chaotic systems, where the master system is given by

$$\begin{aligned} \dot{x}_1 &= f_1(x_1) \\ \dot{x}_2 &= f_2(x_2) \\ \dot{x}_m &= f_m(x_m) \end{aligned} \tag{4}$$

and the controlled slave system is given by

$\dot{y}_1 = g_1(y_1)$
$\dot{y}_2 = g_2(y_2)$
$\dot{y}_x = g_x(y_x)$

(5)

where $x_i y_i = (i = 1, 2 \ldots, n)\, \epsilon R^n$ are state space vectors of the systems, f_n and $g_n: R^n \rightarrow R^n$ are two continuous vector functions and $u_i(i = 1, 2, \ldots, n: Rn \rightarrow Rn$ is a nonlinear control function.

Definition 1:
According to [40] in [4] ff there exists two constants matrices A, B ε R^n and A, B = 0, such that lim∞ $kByi - Axik = 0$ and where k.k is the matrix norm and A,B are scaling matrices, then systems (4) and (5) are said to be in synchronization.

Observation1
The error states in relation to the definition 1 are strictly chosen to satisfy the definition e_{ij} $(i = j = 1, 2, 3 \ldots n)$ are the indices of the error and n refers to the number of dimension of the chaotic system. Hence, we can write $e_{ij} = y_i - x_j$ where y_i, x_i are the slave and master systems respectively. We observe that for $e_{ij} = y_i - x_j$ $(i = j = 1, 2, 3 \ldots n)$, we have $e_{11} = y_1 - x_1$, $e_{22} = y_2 - x_2$, $e_{33} = y_3 - x_3, \ldots e_{nn} = y_n - x_n$ and this is called complete synchronization, in this case there is no switching.

If the error states in relation to definition 1 are redefined such that $e_{ij}(i = j)$ or e_{ij} $(i = j)$ and any of the member in the error dynamical system is interchanged and lim∞ $kByi - Axik = 0$ then, systems (4) and (5) are said to be in switched synchronization state.

Observation 2
If i and j are chosen such that for $e_{ij} = y_i - x_j, (i = j$, then we have
$e_{11} = y_1 -$, $e_{12} = y_1 - x_2$, $e_{13} = y_1 - x_3$, $e_{14} = y_1 - x_4 \ldots e_{1n} = y_1 - x_n$,
$e_{21} = y_2 - x_1, e_{22} = y_2 - x_2, e_{23} = y_2 - x_3$, $e_{24} = y_2 - x_4 \ldots e_{2n} = y_2 - x_n$,
$e_{31} = y_3 - x_1, e_{32} = y_3 - x_2, e_{33} = y_3 - x_3$, $e_{34} = y_3 - x_4 \ldots e_{3n} = y_3 - x_n$,
$e_{41} = y_4 - x_1, e_{42} = y_4 - x_2, e_{43} = y_4 - x_3$, $e_{44} = y_4 - x_4 \ldots e_{4n} = y_4 - x_n$,
$e_{m1} = y_m - x_1, e_{m2} = y_m - x_2, e_{m3} = y_m - x_3, \ldots e_{mn} = y_m - x_n$
for m dimensions of the slave systems and n dimensions of the master systems. We comment here also that there are various switchings that can be formulated in addition to the above.

From system (4), we let $x = x_1, y = y_1, z = z_1$ and $w = w_1$ for the master system and $x = x_2, y = y_2, y = z_2$ and $w = w_2$ for the slave system, and we chose the following switching.

IV. DESIGN OF CONTROLLERS

Let the master system be
$\dot{x}_1 = a_1(y_1 - x_1)$
$\dot{y}_1 = c_1 x_1 + y_1 - x_1 z_1 - w_1$
$\dot{z}_1 = x_1 y_1 - b_1 z_1$

$\dot{w}_1 = d_1 y_1 z_1$

(6)

and the slave system as

$\dot{x}_2 = a_2(y_1 - x_2 + w_2) + u_1$
$\dot{y}_2 = d_2 x_2 - x_2 z_2 + c_2 y_2 + u_2$
$\dot{z}_2 = x_2 y_2 - b_2 z_2 + u_3$

$\dot{w}_2 = y_2 z_2 + e w_2 + u_4$

(7)

Be the slave system, where u_1, u_2, u_3 and u_4 are the set of nonlinear controllers. The switches are chosen as:
Case 1: $\dot{e}_{13} = x_2 - z_1, \dot{e}_{22} = y_2 - y_1, \dot{e}_{34} = z_2 - w_1, \dot{e}_{41} = w_2 - x_1$
Case 2: $\dot{e}_{12} = x_2 - y_1, \dot{e}_{23} = y_2 - z_1, \dot{e}_{34} = z_2 - w_1, \dot{e}_{41} = w_2 - x_1,$
Case 3: $\dot{e}_{14} = x_2 - w_1, \dot{e}_{23} = y_2 - z_1, \dot{e}_{31} = z_2 - x_1, \dot{e}_{42} = w_2 - y_1$
Case 4: $\dot{e}_{11} = x_2 - x_1, \dot{e}_{21} = y_2 - x_1, \dot{e}_{33} = z_2 - z_1, \dot{e}_{44} = w_2 - w_1$
Case 5: $\dot{e}_{14} = x_2 - w_1, \dot{e}_{21} = y_2 - x_1, \dot{e}_{34} = z_2 - w_1, \dot{e}_{43} = w_2 - z_1$

Case 1
Using the notations in comment 2 and differentiating the error dynamics, we have

$\dot{e}_{13} = \dot{X}_1 - \dot{X}_2$
$\dot{e}_{22} = \dot{y}_2 - \dot{y}_1$
$\dot{e}_{34} = \dot{Z}_2 - \dot{W}_1$
$\dot{e}_{41} = \dot{W}_2 - \dot{X}_1$

(8)

And by substituting
$\dot{e}_{13} = -a_2 e_{13} + a_2 e_2 + a_2(y_1 - z_1) - x_1 y_1 + b z_1 + w_2 + u_1$
$\dot{e}_{22} = 2c_2 e_{22} + e_{13} d_2 - z_2(e_{13} + z_1) + y_1(c_2 - 1) + z_1(d_2 + x_1) - c_1 x_1 + w_1 - c_2 e_{22} + u_2$
$\dot{e}_{34} = -b_2 e_{34} - e_{22} + e_{22}(x_2 + 1) + x_2 y_1 - b_2 w_1 - d_1 y_1 z_1 + u_3$
$\dot{e}_{41} = -e e_{41} + 2e e_{41} - e_{34} + e_{34}(y_2 + 1) + y_2 w_1 +$

$x_1(e+a_1) - a_1y_1 + u_4$ (9)

With error dynamics presented above, if appropriate u_1, u_2, u_3 and u_4 are chosen such that the system is stable and unchanged, then asymptotic stabilization would be realized leading to synchronization of the system. If $e_{13} = \eta_1$, its time derivative is $\dot{\eta}_1 = \dot{e}_{13}$ and we can stabilize the first part of (9), using the Lyapunov function, as

$$v_1 = \frac{1}{2}\eta_1^2 \quad (10)$$

whose time derivative is

$$\dot{v}_1 = \eta_1\dot{\eta}_1 \quad (11)$$

By substituting for $\dot{\eta}_1$ from (9) we have

$$\dot{v}_1 = \eta_1(-a_2e_{13} + a_2e_2 + a_2(y_1 - z_1) - x_1y_1 + bz_1 + w_2 + u_1) \quad ($$

choosing $e_{22} = \alpha_1(\eta_2)$ as a virtual controller,

$$u_1 = -a_2(y_1 - z_1) + x_1y_1 - bz_1 + k\eta_1 \quad (13)$$

so that

$$\dot{v}_1 = -(a_2 - k)\eta^2 \quad (14)$$

which is $\leq 0 \; for \; a_2 > 0, k \leq 0$. Thus the subsystem is negative definite and assymptotically stable. Since the error between e_{22} and $\propto_1(z_2)$ is estimative as $\dot{\eta}_2 = e_{22} - \propto_1(\eta_1)$ and $\propto_1(\eta_2) = 0$, the $(\eta_1\eta_2)$ subsystem as

$$\dot{\eta}_1 = -(a2 - k)\eta_1 + a2\eta_2$$

$$\dot{e}_{22} = 2c_2e_{22} + e_{13}d_2 - z_2(e_{13} + z_1) + y_1(c_2 - 1) + z_1(d_2 + x_1) - c_1x_1 + w_1 - c_2e_{22} + u_2 \quad (15)$$

Westablize (15) by discribing the second Lyapunovfunction as

$$v_2 = v_1 + \frac{1}{2}\eta_1^2 \quad (16)$$

Whose time derivative is

$$u_2 = -2c_2e_{22} + z_2(e_{13} + z_1) - y_1(c_2 - 1) - z_1(d_2 + x_1) + c_1x_1 - w_1 + k\eta_2 \quad (18)$$

So that

$$\dot{v}_2 = -(a_2 - k)\,\eta_1^2 - (c_2 - k)\,\eta_2^2 \quad (19)$$

which is $\leq 0 \; for \; a_2 > 0, c_2 > 0, k \leq 0$. Thus the subsystem is negative definite and assymptotically stable. Since the error between $e_{13}$and $\propto_2(\eta_1)$ is estimative as $\eta_1 = \propto_{13} - \propto_1(\eta_1) and \propto_2(\eta_1) = 0, let \; e_{34} \; be \; \eta_3, wecan \; write \; (\eta_1, \eta_2, \eta_3)$ subsystem as

$$\dot{\eta}_1 = -(a_2 - k)\eta_1 + a_2\eta_2$$
$$\dot{\eta}_2 = -(c_2 - k)\eta_2 + d_2\eta_1$$

(20)

$$\dot{\eta}_3 = -b_2e_{34} - e_{22} + e_{22}(x_2 - 1) + x_2y_1 - b_2w_1 - d_1y_1z_1 + u_3$$

We can stabilize (20) by defining the third Lyapunov function as

$$v_3 = v_2 + \frac{1}{2}\eta_3^2 \quad (21)$$

Whose time derivative is

$$\dot{v}_3 = \dot{v}_2 + \eta_3\dot{\eta}_3 \quad (22)$$

By substituting for $\dot{\eta}_3$ in (22), choosing $e_{22} = \propto_3(\eta_2)$ as a virtual controller and choosing

$$u_3 = -e_{22}(x_2 + 1) - x_2y_1 + b_2w_1 + d_1y_1z_1 + k\eta_3 \quad (23)$$

$$\dot{v}_3 = -(a_2 - k)\eta_1^2 - (c_2 - k)\eta_2^2 - (b_2 - k)\eta_3^2 \quad (24)$$

which is ≤ 0 for $a_2 > 0$, $b_2 > 0$, $c_2 > 0$ and $k \leq 0$. Thus, the (η_1, η_2, η_3)subsystem is assymptotically stable. Since the error between $e_{22}$and $\propto_3(\eta_2)$ is estimative as $\eta_2 = e_{22} - \propto_3(\eta_1)$ and $\propto_3(\eta_1) = 0$, let $e_{41}$and $\propto_3(\eta_4)$ $e_{41} = \eta_4$. The $(\eta_1, \eta_2, \eta_3, \eta_4)$ whole system as

$$\dot{\eta}_1 = -(a_2 - k)\eta_1 + a_2\eta_2$$
$$\dot{\eta}_2 = -(a_2 - k)\eta_2 + d_2\eta_1$$
$$\dot{\eta}_3 = -(b_2 - k)\eta_3 - \eta_2 \quad (25)$$

$$\dot{\eta}_4 = -ee_{41} + 2ee_{41} - e_{34} + e_{34}(y_2 + 1) + y_2w_1 + x_1(e + a_1) - a_1y_1 + u_4$$

We can stabilize (25) by defining the fourth Lyapunov function as

$$v_4 = v_3 + \frac{1}{2}\eta_4^2 \quad (26)$$

whose time derivative is

$$\dot{v}_4 = \dot{v}_3 + \eta_4\dot{\eta}_4 \quad (27)$$

By choosing $e_{34} = \propto 4 = (\eta 3)$ as a virtual controller and choosing

We have $u_4 = -2ee_{41} - e_{34}(y_2 + 1) - y_2w_1 - x_1(e + a_1) + a_1y_1 + k\eta_4$ (28)

$$\dot{v}_4 = -(a_2 - k)\eta_1^2 - (c_2 - k)\eta_2^2 - (b_2 - k)\eta_3^2 - (e - k)\eta_4^2 \quad (29)$$

which is $\leq 0 \; for \; a2 > 0, b2 > 0, c2 > 0 \; and \; k \leq 0$ Thus, the whole system $(\eta 1, \eta 2, \eta 3, \eta 4)$ is assymptotically stable. For other cases (2-5), the controllers are as presented in equations (30), (31), (32) and (33) respectively.

$$u_1 = -a_2(z_1 - y_1) + x_1c_1 + y_1 - x_1z_1 - w_1 - w_2 + k\eta_1$$

$$u_2 = -\eta_2(c_1 + 1) + z_2(e_1 + y_1) - z_1(c_2 + b_1) - d_2y_2 + x_1y_1 + k\eta_2$$

$u_3 = -\eta_1(y_2+1) - y_1y_2 + y_1d_1z_1 + b_2w_1 + k\eta_3$

$u_4 = -\eta_4(e+1) - \eta_3(y_2+1) - y_2w_1 - x_1(e-a_1) + a_1y_1 + k\eta_3$ (30)

$u_1 = -a_2(z_1 - w_1) - w_2 + d_1y_1z_1 + k\eta_1$

$u_2 = -\eta_2(c_2+1) + \eta_3(x_2+1) - x_2(d_2 - x_1) - z_1(c_2+b_1) + x_1y_1 + k\eta_2$

$u_3 = -\eta_2(c_2+1) - x_1(a_1 - b_2) - x_2z_1 + a_1y_1 + k\eta_3$

$u_4 = -\eta_4(e+1) - \eta_3(y_2+1) - x_1(y_2 - c_1 + z1 - y1e - 1 - w1 + k\eta 4$ (31)

$u_1 = -a_1(y_1 - x_1) - w_2 + k\eta_1$

$u_2 = -\eta_2(c_2+1) + z_2(e_1 + x_1) - x_1(d_2 - c_2 - a1 + a1y1 + k\eta 2$

$u_3 = -\eta_4 - x_2y_2 + x_1y_1 + b_2z_1 - b_1z_2 + k\eta_3$

(32)

$u_4 = -\eta_4(e-1) - \eta_3(y_2+1) - z_1y_2 + z_1d_1y_1 - ew_1 + k\eta_4$

$u_1 = -a_2(x_1 - w_1) - w_2 + d_1y_1z_1 + k\eta_1$

$u_2 = -\eta_2(c_2+1) + x_1(c_2 + a_1) + z_2(\eta_1 + w1 + a1y1 - w1d2 + k\eta 2$

$u_3 = -\eta_4 - \eta_4d_1y_1 - x_2y_2 + b_2w_1 + d_1y_1w_2 + k\eta_3$

$u_4 = -\eta_4(e+1) - \eta_3(y_2+1) - y_2w_1 - z_1(e+b_1) + x_1y_1 + k\eta_4$ (33)

V. NUMERICAL SIMULATION

In order to verify the effectiveness of controllers derived above and those in the equation (30 – 33) we present our numerical simulation. We used the Runge-Kutta simulation tool. In all the figures presented, the hyperchaotic Lorenz system (drive) parameters selected remain constant at $a_1 = 10$, $b_1 = 8/3$, $c_1 = 28$, $d_1 = 2.25$, with initial condition values $x_1 = 0.1$, $y_1 = 0.1$, $z_1 = 0.1$, $w_1 = 0.1$, the hyperchaotic Chen system (response), parameters selected remain constant at $a_2 = 35$, $b_2 = 3$, $c_2 = 12$, $d_2 = 7$, $e_2 = 0.5$ and variable parameters chosen as $x_2 = 0.5$, $y_2 = 0.01$, $z_2 = 0.8$, $w_2 = 0.5$. the step size was maintained at H = 0.005 and t = 10. The synchronization of the slave system with the master system at each of the state variable is presented in (Figures 3) for states e_1, e_2, e_3 and e_4 respectively, while the fifth graph in this figure shows the result for the combined states for case 1. In each of these figures, synchronization took place when each of the controllers u_1, u_2, u_3 and u_4 was activated at $t \geq 10$. Using the same parameters, and making use of our results for other cases 2, 3, 4 and 5 for the non-identical hyperchaotic systems unified and periodically converged to zero as shown in the figures 4 as time tends to infinity which signifies that the global synchronization between systems (5) and (6) has been achieved.

VI. CONCLUSION

We have analyzed and validated the possibility of the multiswitchingsynchronization of the non-identical hyperchaotic Lorenz (drive) and that of Chen's (response) systems based on integrator back stepping technique. We extended the usual master-slave synchronization scheme for low order chaotic systems to study the synchronization of this higher order systems on one side, and provided various switches in the design of the controllers. Each of the 4-dynamical states was successfully synchronized. The synchronization of each of the switches in other cases was also successful. By implication, electronic information can be hidden in any or all of this 4D hyperchaotic system and such information can be locked up in any of the states in each of the cases, with at least four different switch codes.

Such information can be transferred, communicated and retrieved by applying the control inputs for each or all the dynamical states and respective switches. This makes the information more secure not only because of the hyperchaos status of the system in consideration, but also the several switches that must be unlocked to retrieve the information. Our numerical results confirm the effectivenessof the analytical technique and we belief that they are observable in laboratory experiments.

VII. ACKNOWLEDGMENT

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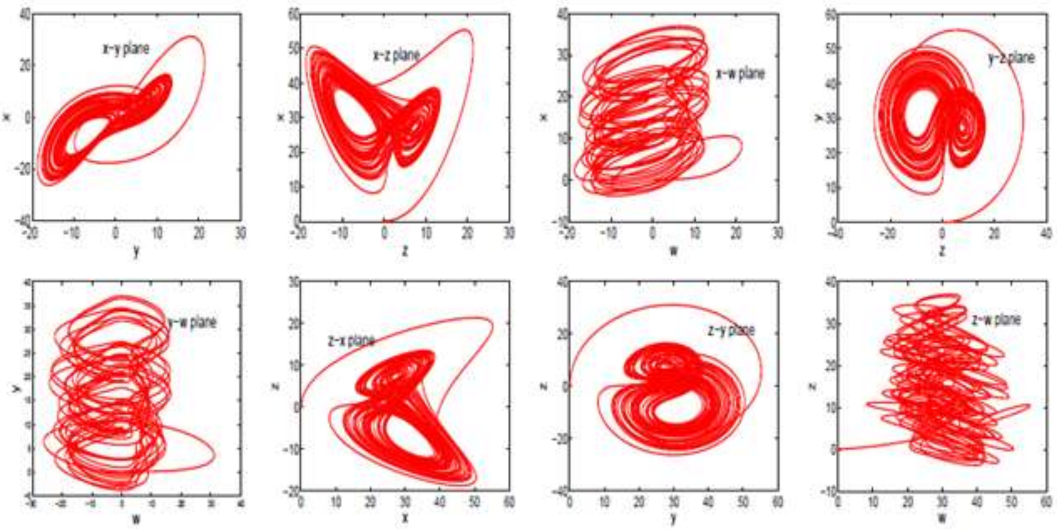

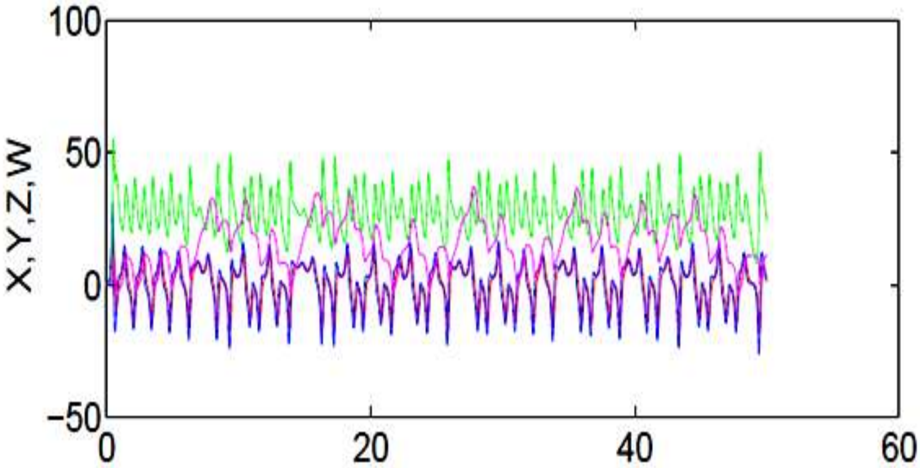
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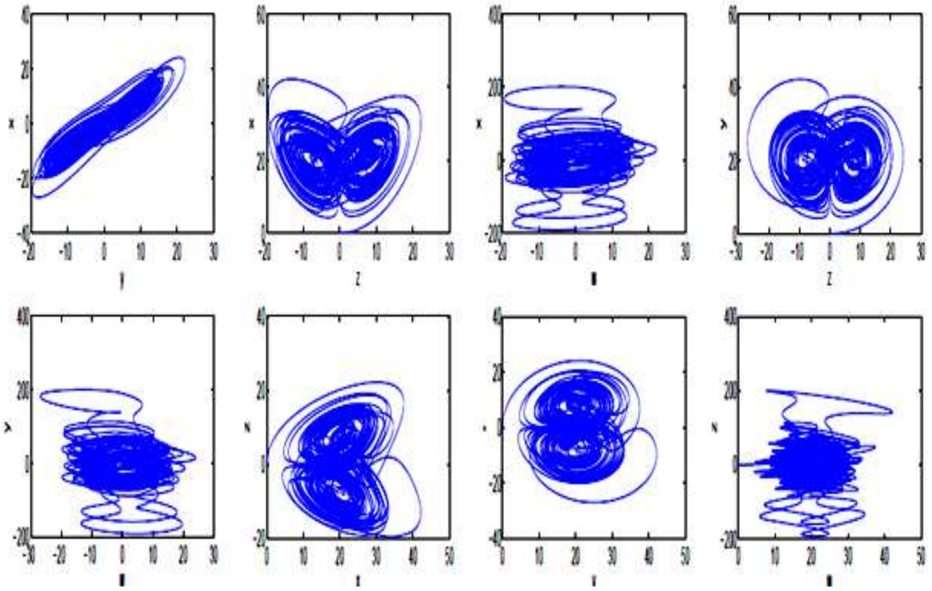

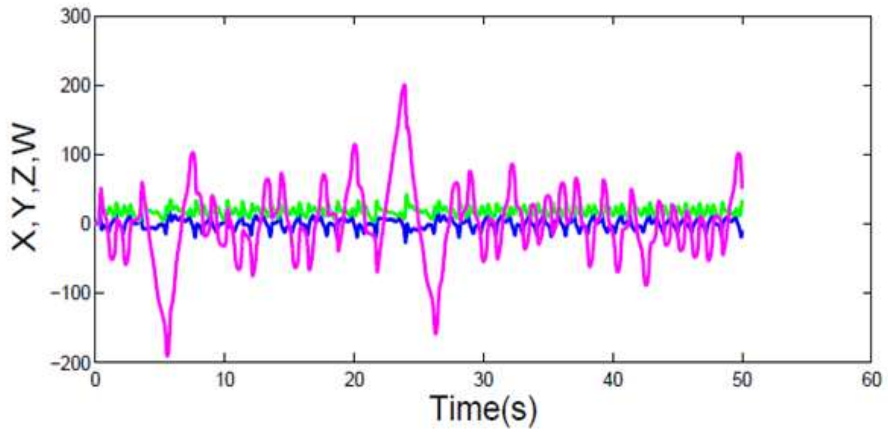
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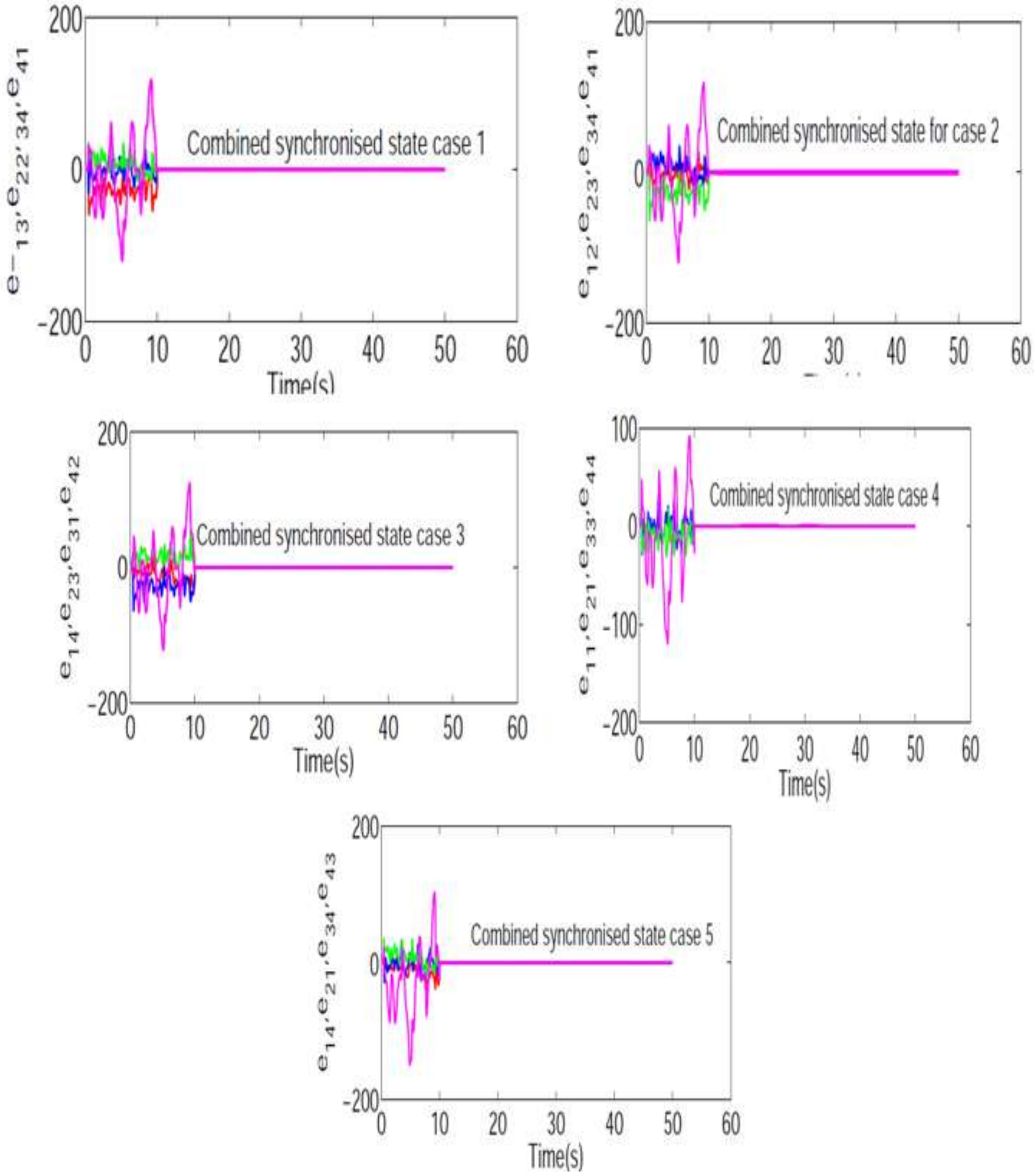

Combined synchronised state case 1
Combined synchronised state for case 2
Combined synchronised state case 3
Combined synchronised state case 4
Combined synchronised state case 5
Time(s)