

# Applications of Runge-Kutta-Fehlberg Method and Shooting Technique for Solving Eyring-Powell Fluid through a Channel with Thermal Radiation.

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**ABSTRACT:** In this paper, we examine the Eyring-powell model in the presence of thermal radiation and heat source and chemical reaction. The partial differential equations governing the fluid flow are converted into system of Non-linear ordinary differential equation is then solved numerically using the Runge-Kutta-Fehlberg method with shooting technique. The numerical method used shows that the results are in agreement and is consistent with published result. The accuracy of the proposed method is higher than other analytical solutions; hence proposed method is efficient and in agreement with existing literature.  
**Indexed Terms:** Eyring-powell model, non-Newtonian fluid, similarity transformation, Runge-Kutta-Fehlberg fourth fifth order numerical method, shooting method.

## I. INTRODUCTION

Recently, Squeezing flow of Newtonian and Non-Newtonian fluids has gained considerable attention from researchers due to its importance and its application in Science, engineering and Technology. Such flows are induced by approaching parallel surfaces on disk in relative motion. The first author to work on this was Stefan [1], did pioneering work on the squeezing flow by initiating lubrication approach. Domairy and Aziz [2] employed He's homotopy perturbation method to consider the suction and injection effects on the flow of electrically conducting viscous fluid squeezed between parallel disks. Hayat et al. [3,4] made an improvement on the work analyzed in [2] to analyze the squeezing flow of non-Newtonian fluids by taking second grade and micropolar

models. While Qayyum et al [5] Investigated the unsteady squeezing flow of viscoelastic Jeffery fluid between parallel disks and discussed the porosity and squeezing effects on the velocity profile. Although, considerable authors have studied the heat transfer characteristics of viscous fluid in squeezing flow between two parallel surfaces, Squeezing flow and heat transfer over a porous surface was investigated by Mahmood et al.[6] Mustafa et al.[7] Considered the Combine effects of heat and mass transfer in a viscous fluid squeezed between two parallel plates. Also of recent, Bahadir and Abasov [8] used the finite difference scheme to analyze and obtain numerical solution of hydromagnetic steady flow and heat transfer in a viscous fluid squeezed between parallel disks. In practical the no slip boundary conditions is known to be one of the core concepts of fluid mechanics, i.e. the assumption that when a liquid flows over a solid surface, the liquid molecules adjacent to the solid are stationary relative to solid [9]. In recent year Adesanya et al [10] applied adomian decomposition method to the analysis of effect of couple stresses on hydromagnetic Eyring-Powell fluid flow through a porous channel. Some examples of Non-Newtonian fluids are Powell-Eyring fluids . Here are some examples of research done in this area. Analysis of boundary layer flow and heat transfer due a Powell-Eyring fluid on a moving permeable surface in a parallel free stream and analysis of governing nonlinear differential equations using Keller box method and also investigated effects of some parameters on flow field were considered by Jalil et al. [11]. Analysis of mixed convection

unsteady boundary layer flow due a rotating Powell- Eyring fluid over a rotating cone considering combined effects of heat and mass transfer were considered by Nadeem and Saleem [12]. HAM was the semi-analytic method employed to dissolve the governing nonlinear differential equations and then, they investigated the effects of several parameters on velocity, temperature, concentration, skin friction coefficient, Sherwood number and Nusselt number. Agbaje et al. [13] used a numerical method for unsteady non-Newtonian Powell- Eyring nanofluid flow on a shrinking sheet considering the heat generation and thermal radiation on the fluid flow. The analysis of boundary layer flow of non-Newtonian Eyring-Powell fluid on a linear stretching sheet using collocation method and transformation of its nonlinear and PDEs equations to ordinary and ODEs equations using a similar transformation method and investigating the behavior of velocity profile were carried out by Rahimi et al. [14]. Simulation the steady 3D magnetohydrodynamics flow due a Powell- Eyring nanofluid with convective and the nano particles mass flux conditions and solving governing equations using bvp4c method were carried out by Khan et al. [15]. Taking into consideration the issues analyzed and documented in this section, this article also studies the heat and mass transfer in an unsteady two- dimensional squeezing flow of magnetohydrodynamic (MHD) radiative non-Newtonian Eyring-Powell fluid through a channel with thermal radiation and heat generation/absorption. The analysis and solving of the non-linear ordinary differential equations governing the flow using Runge-Kutta-Fehlberg fourth fifth order method with shooting technique is considered.

Mustapha and Salau [16] worked on the solution of heat and mass transfer analysis in nanofluid flow using Homotopy perturbation method. Adomian decomposition method was also employed by Mustapha and Salau [17] for solving

heat transfer analysis for squeezing flow of Nanofluid in parallel disks. A comparative solution of heat transfer analysis for squeezing flow between parallel disks was also investigated by Mustapha and Salau [18] using the mentioned method alongside Adomian decomposition method. In [19] Mustapha et al gave detailed insight to Semi-analytic solution of Riccati equation using Homotopy perturbation method and Adomian decomposition method (ADM). Mustapha and Salau [20] also used HPM to get a series solution of Euler-Bernoulli Beam subjected to a concentrated load.

## II. MATHEMATICAL MODEL

Considering an incompressible non-Newtonian Eyring-Powell fluid, the constitutive equation is given by [27] as

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \left[ \frac{1}{\beta|\mathbf{A}_1|} \sinh^{-1} \left( \frac{1}{c} |\mathbf{A}_1| \right) \right] \mathbf{A}_1 \quad (1)$$

Where  $\mathbf{T}$  is cauchy stress tensor,  $p$  is the pressure,  $\mathbf{I}$  is unit tensor,  $\mu$  is dynamic viscosity,  $\beta$  and  $c$  are characteristics of the Eyring-Powell fluid with dimensions  $\text{pascal}^{-1}$  and  $\text{second}^{-1}$ , respectively,  $\mathbf{A}_1$  is the Rivlin-Ericksen tensor defined as

$$\mathbf{A}_1 = (\nabla\mathbf{V}) + (\nabla\mathbf{V})^T \quad (2)$$

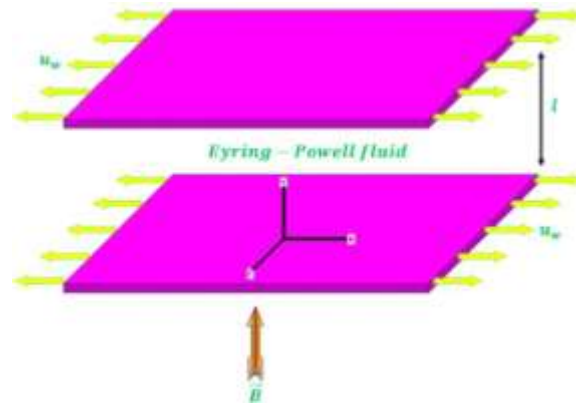
$$|\mathbf{A}_1| = \sqrt{\frac{1}{2} \text{tr}(\mathbf{A}_1^2)} \quad (3)$$

here  $\nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$ ,  $\mathbf{V}$  is the velocity of the flow field and  $\mathbf{T}$  is the transpose of the matrix. Taking the second-order approximation of  $\sinh^{-1}$  function and neglecting higher term we have

$$\sinh^{-1} \left( \frac{1}{c} |\mathbf{A}_1| \right) \cong \frac{|\mathbf{A}_1|}{c} - \frac{|\mathbf{A}_1|^3}{c^3}, \quad \left| \frac{|\mathbf{A}_1|}{c} \right| \ll 1 \quad (4)$$

## III. PHYSICAL MODEL DESCRIPTION

The geometry of the physical model is shown in fig. 1 below



We consider unsteady MHD two-dimensional squeezing flow of an incompressible Eyring-Powell fluid between two infinite plates separated by a distance  $a(t) = \pm l\sqrt{1-\alpha t}$ . Here  $l$  is the initial distance between the plates at time  $t$  and  $\alpha$  is the characteristic parameter of the squeezing motion of the plate with dimension of the inverse time. For  $\alpha > 0$ , both plates squeeze until they meet at  $t = 1/\alpha$  and  $\alpha < 0$  indicates that the plates are separated. Take a Cartesian coordinate system  $(x, y)$  where the central axis of the channel is taken

as the  $x$ -axis and the  $y$ -axis is perpendicular to it. A uniform magnetic field  $B = \frac{B_0}{\sqrt{1-\alpha t}}$  is applied normally to the wall, in the presence of thermal radiation and heat generation/absorption. Further assuming the induced magnetic field is negligible due to low Reynolds numbers. The governing equations of mass, momentum, energy and mass transfer describing the unsteady two-dimensional flow under the above assumptions can be written as [21]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (5)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \left(v + \frac{1}{\rho\beta c}\right) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{3\rho\beta c^3} \frac{\partial}{\partial x} \left\{ \left(2 \left(\frac{\partial u}{\partial x}\right)^2 + 2 \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right) \frac{\partial u}{\partial x} \right\} - \frac{1}{6\rho\beta c^3} \frac{\partial}{\partial y} \left\{ \left(2 \left(\frac{\partial u}{\partial x}\right)^2 + 2 \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right) \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \right\}$$

$$- \frac{\sigma B_0^2}{\rho(1-\alpha t)} u(6)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \left(v + \frac{1}{\rho\beta c}\right) \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{1}{6\rho\beta c^3} \frac{\partial}{\partial x} \left\{ \left(2 \left(\frac{\partial u}{\partial x}\right)^2 + 2 \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right) \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \right\} - \frac{1}{3\rho\beta c^3} \frac{\partial}{\partial y} \left\{ \left(2 \left(\frac{\partial u}{\partial x}\right)^2 + 2 \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right) \frac{\partial v}{\partial y} \right\} (7)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{16\sigma^* T_0^3}{3\rho C_p \kappa^*} \frac{\partial^2 T}{\partial y^2} + \frac{Q^*}{\rho C_p} (T - T_0) \quad (8)$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) - \frac{k_1}{1-\alpha t} (C - C_0) \quad (9)$$

Subjected to the below boundary conditions

$$u = 0, \quad v = v_w = \frac{da(t)}{dt}, \quad T = T_0, \quad C = C_0, \quad \text{at } y = a(t)$$

$$u = \frac{\partial u}{\partial y} = \frac{\partial T}{\partial y} = \frac{\partial C}{\partial y} = 0, \quad \text{at } y = 0. (10)$$

The velocity the velocity components are  $u$  and  $v$  in  $x$  and  $y$  directions respectively, the kinematic viscosity is  $\nu$ , the fluid density is  $\rho$ , the electrical conductivity of the fluid is denoted as  $\sigma$ , the fluid temperature is  $T$ ,  $\kappa$  is the thermal conductivity of the fluid,  $C_p$  is the specific heat capacity,  $\sigma^*$  is the Stefan-Boltzman constant,  $\kappa^*$  is the mean absorption coefficient,  $Q^*$  is the uniform volumetric heat generation/absorption coefficient,  $C$  is the concentration species of the fluid,  $D_m$  the mass diffusion,  $k_1$  is the reaction rate, referenced fluid temperature and concentration species of the fluid flow are  $T_0$  and  $C_0$  respectively.

Introducing the following non-dimensional quantities:

$$\eta = \frac{y}{l\sqrt{1-at}}, \quad u = \frac{\alpha x}{2(1-at)} f'(\eta), \quad v = -\frac{al}{2\sqrt{1-at}} f(\eta), \quad \theta = \frac{T-T_0}{T_1-T_0}, \quad \phi = \frac{C-C_0}{C_1-C_0} \quad (11)$$

Using the non-dimensional variables in equation (11) the continuity equation (5) is identically satisfied and equations (6) and (7) after eliminating the generalized pressure gradient from the resulting equation results to:

$$(1 + \Gamma)f^{iv} - S(\eta f'''' + 3f'' + f'f'' - ff'''' - \Gamma\delta(2f''(f''')^2 + (f'')^2 f^{iv}) - M^2 f'' = 0. \quad (12)$$

Also, substituting equation (11) in equations (8) - (10) reduces to the forms below:

$$\left(1 + \frac{4}{3}R\right)\theta'' + PrS(f\theta' - \eta\theta' + Q\theta) = 0, \quad (13)$$

$$\phi'' + ScS(f\phi' - \eta\phi') - Sc\gamma\phi = 0, \quad (14)$$

$$f(0) = 0, f''(0) = 0, \quad \theta'(0) = 0, \phi'(0) = 0, \quad (15)$$

$$f'(1) = 0, f(1) = 1, \quad \theta(1) = 1, \phi(1) = 1.$$

where prime is the differentiation with respect to  $\eta$ ,  $\Gamma$  and  $\delta$  are the fluid parameters,  $S$  is the squeezing value,  $M$  is the Hartman number, the radiation parameter is  $R$ ,  $Pr$  is the prandtl number,  $Q$  is the heat generation parameter for  $Q > 0$  or heat absorption parameter for  $Q < 0$ ,  $Sc$  is the Schmidt number and  $\gamma$  is the chemical reaction parameter. These parameters are given below as:

$$\Gamma = \frac{1}{\mu\beta c}, \quad \delta = \frac{\alpha^2 x^2}{8c^2 l^2 (1-at)^3}, \quad S = \frac{al}{2\nu}, \quad M^2 = \frac{\sigma B_0^2 l^2}{\mu}, \quad R = \frac{4\sigma^* T_0^3}{\kappa k^*}, \quad (16)$$

$$Pr = \frac{\mu C_p}{\mu}, \quad Q = \frac{2Q^*(1-at)\beta_1}{\alpha\rho C_p}, \quad Sc = \frac{\nu}{D_m}, \quad \gamma = \frac{l^2 k_1}{\nu}.$$

It is important to consent that the fluid parameter  $\delta$  is a function of  $x$ , and its values varies locally throughout the flow motion. Also it is worthwhile to mention that the squeeze value(number) describes motion of the manifolds(plates)

(the movement of the Plate apart corresponds to  $S > 0$ , while  $S < 0$  corresponds to the plates collapsing together, the so-called squeezing flow). Furthermore,  $\gamma > 0$  indicates the destructive chemical reaction and  $\gamma < 0$  represents the generative chemical reaction.

The physical properties of interest are the skin-friction coefficient  $C_f$ , local Nusselt number  $Nu$  and local Sherwood number  $Sh$  which can be expressed by the following:

$$C_f = \frac{\tau_w}{\rho\nu_w^2}; \tau_w = \left(\mu + \frac{1}{\beta c}\right) \frac{\partial u}{\partial y} - \frac{1}{6\beta c^3} \left(\frac{\partial u}{\partial y}\right)^2 \Big|_{y=a(t)}, \quad (17)$$

$$Nu = \frac{lq_w}{\kappa(T_1 - T_0)}; q_w = -\kappa \frac{\partial T}{\partial y} - \frac{16\sigma^* T_0^3}{3k^*} \frac{\partial T}{\partial y} \Big|_{y=a(t)}, \quad (18)$$

$$Sh = \frac{lJ_w}{D_m(C_1 - C_0)}; J_w = D_m \frac{\partial C}{\partial y} \Big|_{y=a(t)} \quad (19)$$

In terms of equation (11), the dimensionless forms of equations (17 - 19) are given by

$$Re_x \frac{l^2}{x^2} C_f = \left\{ (1 + \Gamma)f''(1) - \frac{\Gamma\delta}{3} [f''(1)]^3 \right\} \quad (20)$$

$$\sqrt{1-at}Nu = -\left(1 + \frac{4}{3}R\right)\theta'(1), \sqrt{1-at}Sh = -\phi'(1), \quad (21)$$

Where  $Re_x = \frac{alx\sqrt{1-at}}{2\nu}$  is the local squeeze Reynolds number.

#### IV. SHOOTING METHOD

Shooting method is a technique that can be used to solve varieties of BVP. In this method by choosing the arbitrary value for derivatives of desired function in starting time and converting BVP into initial value problem (IVP) we were able to apply the aforementioned technique, Shooting techniques were extensively dealt with in [24]. In starting time, the method chooses an arbitrary value of the one end of the problem in question (IVP). Solution starts at one end of the BVP and “Shoot” to the arbitrary value of the IVP until the boundary condition at other end hit to its correct value. More insight are given in [22].

In this work, we consider an efficient numerical scheme Runge-Kutta-Fehlberg fourth fifth order Method (RKF45) which as been employed to analyse the flow model in question. In Numerical, the RK–Fehlberg is a very powerful procedure(tool) for solving an ordinary differential equation numerically even including non-linear ordinary differential equations especially the ones that arises in Computational fluid dynamics . This method was introduced by Fehlberg [23] and is considered as the large class of RK techniques. This method is an embedded form of the RK family and is distributed mostly across Mathematical software I.e Maple, Mathematica, Matlab etc .Below is the Butcher tableau for Runge-Kutta-Fehlberg fourth fifth order Method (RKF45).

$$\begin{array}{c|cccccc}
 0 & & & & & & \\
 1 & \frac{1}{4} & & & & & \\
 \hline
 3 & \frac{3}{8} & \frac{9}{32} & & & & \\
 \hline
 12 & \frac{1932}{2197} & \frac{7200}{2197} & \frac{7296}{2197} & & & \\
 13 & & & & & & \\
 \hline
 1 & \frac{439}{216} & -8 & \frac{3860}{513} & \frac{845}{4104} & & \\
 1 & \frac{1}{8} & & \frac{3544}{2565} & \frac{1859}{4104} & \frac{11}{40} & \\
 2 & -\frac{1}{27} & 2 & -\frac{2565}{2565} & \frac{4104}{4104} & -\frac{1}{40} & \\
 \hline
 & \frac{25}{216} & 0 & \frac{1408}{2565} & \frac{2197}{4104} & -\frac{1}{5} & 0
 \end{array}$$

#### V. DISCUSSION OF RESULTS

In this section, we present tables that shows the effect of pertinent parameters on the Eyring-Powell fluid in talk. We considered the effects of these parameters and their direct consequence on the skin-friction coefficient  $C_f$ , local Nusselt number  $Nu$  and local Sherwood number  $Sh$ , and they are tabulated for more insight. Runge-Kutta shooting method (RKSM) was the numerical method used to generate numerical results for fixed values of the parameters  $S, \Gamma, \delta, M, R, Pr, Q, Sc$  and  $\gamma$ , as shown in table 1. and compared with the semi-analytic method used in [25]

Table 1. Convergence of solutions when  $S = 0.5, \Gamma = \delta = \gamma = 0.1, M = R = Pr = Q = 1$  and  $Sc = 0.6$

$\eta$	$f(\eta)$			$\theta(\eta)$			$\phi(\eta)$		
	[25]	RKSM	Error	[25]	RKSM	Error	[25]	RKSM	Error
0	0.00000000	0.00000000	0	1.24436958	1.11635116	$1.28 \times 10^{-1}$	0.95279202	0.97111743	$1.83 \times 10^{-2}$
0.1	0.13586036	0.14340981	$7.55 \times 10^{-3}$	1.24136511	1.11515547	$1.26 \times 10^{-1}$	0.95326834	0.97140872	$1.81 \times 10^{-2}$
0.2	0.27024429	0.28452102	$1.43 \times 10^{-2}$	1.23245465	1.11157316	$1.21 \times 10^{-1}$	0.95469642	0.97228200	$1.76 \times 10^{-2}$
0.3	0.40154944	0.42097009	$1.94 \times 10^{-2}$	1.21793614	1.10561826	$1.12 \times 10^{-1}$	0.95707374	0.97373563	$1.67 \times 10^{-2}$
0.4	0.52791339	0.55026183	$2.23 \times 10^{-2}$	1.19826972	1.09731328	$1.01 \times 10^{-1}$	0.96039646	0.97576714	$1.54 \times 10^{-2}$
0.5	0.64706222	0.66969914	$2.26 \times 10^{-2}$	1.17402303	1.08668785	$8.73 \times 10^{-2}$	0.96466003	0.97837367	$1.37 \times 10^{-2}$
0.6	0.75613223	0.77630688	$2.02 \times 10^{-3}$	1.14579471	1.07377686	$7.20 \times 10^{-2}$	0.96985996	0.98155258	$1.17 \times 10^{-2}$
0.7	0.85145329	0.86674673	$1.53 \times 10^{-2}$	1.11411598	1.05861794	$5.54 \times 10^{-2}$	0.97599285	0.98530227	$9.31 \times 10^{-3}$
0.8	0.92828058	0.93721854	$8.94 \times 10^{-3}$	1.07933042	1.04124834	$3.81 \times 10^{-2}$	0.98305768	0.98962324	$6.47 \times 10^{-3}$
0.9	0.98045878	0.98334111	$2.88 \times 10^{-3}$	1.04145177	1.02170104	$1.98 \times 10^{-2}$	0.99105727	0.99451945	$3.46 \times 10^{-3}$
1	1.00000000	1.00000000	0	0.99999999	1.00000000	$1.0 \times 10^{-8}$	1.00000000	1.00000000	0

Table 2: skin friction coefficient values for different values of  $S, \Gamma, \delta, M$  when  $R = Pr = Q = \gamma = 1$  and  $Sc = 0.1$

$S\Gamma\delta M C_f$				
-3	1	0.01	1	-
3.52021462				
-1	1	0.01	1	-
5.40895086				
0.1	1	0.01	1	-
6.20862518				
1	1	0.01	1	-
6.78475647				
3	1	0.01	1	-
7.88925417				
5	1	0.01	1	-
8.82538866				
5	3	0.01	1	-
15.02750969				
5	5	0.01	1	-
21.0557266				

5	1	0.01	1	-
7.88925417				
5	1	0.01	1	-
7.79660483				
5	1	0.01	1	-
7.69939952				
5	1	0.01	1	-
8.82538866				
5	1	0.01	3	-
9.79204115				
5	1	0.01	5	-
11.45010489				

Table 3: local Nusselt number values for different values of S,R,Pr, andQ when  $\Gamma = M = \gamma = 1.0, \delta = 0.01,$  and  $Sc = 0.1$

$SRPrQ - \left(1 + \frac{4}{3}R\right)\theta'(1)$				
-3	0.1	1	0.1	-0.33677721
-1	0.1	1	0.1	-0.10315315
0.1	0.1	1	0.1	0.00997186
1	0.1	1	0.1	0.09740972
3	0.1	1	0.1	0.28015679
5	0.1	1	0.1	0.45200094
5	0.2	1	0.1	0.45730753
5	0.3	1	0.1	0.46156474
5	0.1	1	0.1	0.4520094
5	0.1	1.3	0.1	0.5677375
5	0.1	1.5	0.1	0.63963515
5	0.1	1	-0.2	-0.65712179
5	0.1	1	-0.1	-0.3594576
5	0.1	1	0	0
5	0.1	1	0.1	0.4520094
5	0.1	1	0.2	1.05262913

Table 4: local Sherwood number values for different values of S,R,Pr, andQ when  $\Gamma = M = \gamma = 1.0$  and  $Sc = 0.1$

$SSc\gamma - \phi'(1)$				
-3	0.1	1	-	
0.09906042				
-1	0.1	1	-	
0.09744372				
0.1	0.1	1	-	
0.09673403				
5	0.1	1	-	
0.09433714				
5	0.3	1	-	
0.25435158				
5	0.5	1	-	
0.38520541				
5	0.1	-0.8	0.07997819	
5	0.1	-0.4	0.03946034	
5	0.1	0	0	
5	0.1	0.4	-	
0.03845162				
5	0.1	0.8	-	

0.07594033

## VI. CONCLUSIONS

In this paper, a study on the heat and mass transfer analysis of magnetohydrodynamic (MHD) unsteady squeezing flow of Eyring-Powell fluid through a channel with thermal radiation and heat generation/absorption, numerical technique is used to compare with the semi-analytic method used in [25]. There was a comparison between numerical results generated and existing literature which shows excellent agreement. In addition, the method in question used converges faster and this shows it's a very efficient numerical method. The results were tabulated. We can see that the numerical results presented from the tables in Discussion of results section are accurate.

### Conflict of Interest

The authors hereby consent that as pertaining to this research work that there is no conflict of interest.

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