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Analysis and Computation of Lifting Force around Joukowsky Airfoil

Garba, Mohammed, Prof. N.H. Manjak, Prof. Sabo Hamma and Dr. M. S. Adamu

1. Department of Science Education, Bauchi State University Gadau, Nigeria.
2, 3,4. Department of Mathematical Science, Abubakar Tafawa Balewa University Bauchi, Nigeria.

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ABSTRACT
One of the more important potential flow results obtained using conformal mapping are the solutions of the potential flows past a family of airfoil shapes known as Joukowsky foils. Solving problem of fluid flow around an airfoil is a highly complex task. However, reducing the problem to the two-dimensional ideal fluid flow allows one to employ techniques of complex variables, in particular utilizing the geometric properties of conformal mappings. In this study, we analysed the two dimensional ideal fluid flow around a circular cylinder obtained by the superposition of simple elementary flows, then relate the solution (complex potential) to uncambered airfoil by means of Joukowsky transformation, and it was found that the circulation and the lift force around a circular cylinder in the z plane remained unchanged around transformed uncambered airfoil in the w plan. MATLAB software was used to visualize the streamlines around the circular cylinder and the corresponding Joukowsky airfoil. The lift force was calculated at different angles of attack and it was found that the lift force is strongly dependent to the angle of attack in a linear proportion.

Keywords and Phrases: Conformal Map, Joukowsky Transformation, Airfoil, Inviscid, Incompressible.

I. INTRODUCTION

Many problems in Mathematics are difficult to solve in their original geometric form. If the physical problem can be represented by a complex functions but geometric structure becomes inconvenient then by an appropriate mapping , it can be transformed to a problem with much more convenient geometry (Sa Pai, 2020).

In fluid dynamics, a field of significant importance is the study of airfoils**.** Solving problem of fluid flow around an airfoil is a highly complex task. However, reducing the problem to the two-dimensional fluid flow allows one to employ techniques of complex variables, in particular utilizing the geometric properties of conformal mappings given by

$$w = f(z) \tag{1}$$

Where $z = x + iy$ and $w = u + iv$. Using this technique, in particular Joukowsky transformation, the fluid flow around the geometry of an airfoil can be model as the flow around a cylinder whose symmetry simplifies the needed computations (Kapania et al, 2008).

In this pepper, we are to show geometrically via Joukowsky transformation the corresponding geometry in fluid dynamics, specifically, to exploit Joukowsky transformation to analyse the geometric nature of Ideal fluid flow.

Joukowsky transformation is a conformal mapping, that plays an important role in the study of flows around airfoils, given by

$$w = f(z) = z + \frac{\lambda^2}{z} \tag{2}$$

where w is the function in the transformed plane and λ is the parameter of the transformation that determines the resulting shape of the transformed function (Maloneka & De Almeida, 2010).

The flow around airfoil can be solved by solving the flow around circular cylinder, then relate the solution to the airfoil by means of Joukowsky transformation, equation (2)

Joukowsky transformation; is used because it has the property of transforming circles in the z plane into shapes that resemble airfoils in the w plane (Benson & Thomas, 1996).

A lifting flow around a circular cylinder is generated by the superposition of three elementary flows, namely a uniform flow, source-sink flow and vortex flow (Nyadwi, 2018). This requires that the velocity potential and stream function be expressed as a complex function, just as the airfoil shape

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must also be defined using complex variables, given by

$$w(z) = \varphi + i\psi \quad (3)$$

(Bear & Jacob, 1972)

If, moreover, $\psi(x, y)$ denotes a harmonic conjugate of $\varphi(x, y)$, the velocity vector is tangent to a curve $\psi(x, y) = c$.

The curve

$$\psi(x, y) = c \quad (4)$$

is called the streamline of the flow, and the function ψ is the stream function. In particular, a boundary across which fluid cannot flow is a streamline (Brown & Churchill, 2009).

Theorem 1: Consider the lifting flow past a cylinder, then its complex potential defined in equation (3) can be written as follows:

$$w(z) = V_\infty\left(z + \frac{R^2}{z}\right) + i\frac{\Gamma}{2\pi}\ln(z) \quad (5)$$

Therefore (5) decompose into,

$$w(z) = V_\infty\left(r + \frac{R^2}{r}\right)\cos\theta - \frac{\Gamma}{2\pi}\theta + i\left[V_\infty\left(r - \frac{R^2}{r}\right)\sin\theta + \frac{\Gamma}{2\pi}\ln(r)\right]$$

Hence,

$$w(z) = \varphi(r\,\theta) + i\psi(r\,\theta)$$

and the stream function ψ of the combined flow is given by the equation

$$\psi = V_\infty r\sin\theta\left(1 - \frac{R^2}{r^2}\right) + \frac{\Gamma}{2\pi}\ln r \quad (6)$$

Once the flow around a rotating cylinder has been solved, the lift force can be computed by means of the Kutta-Joukowsky theorem. (Prachi , 2020)

Theorem 2: Kutta-Joukowsky theorem, states that the lift per unit span L' is directly proportional to the circulation Γ. For the proof see Panton 2013.

Circulation Γ, is a conceptual tool that relates the lift on an object to the nature of the fluid flow around it. Specifically, circulation is related to vorticity on any open surface bounded by the airfoil curve, using Stoke's theorem

$$\Gamma = \oint \vec{V}\cdot d\vec{s} = \iint\left(\nabla \times \vec{V}\right)\cdot d\vec{S} = \iint \vec{w}\cdot d\vec{S} \quad (7)$$

For the lifting flow around a cylinder, the only source of vorticity comes from the vortex flow, which has an infinite vorticity at the origin but zero vorticity at every other point. This point source of vorticity leads to the finite circulation Γ. However, Γ is also the strength of a vortex flow. Therefore, for a flow, there are an infinite number of arbitrary Γ values, each corresponding to a deferent flow solution satisfying Laplace equation.

Corollary : When the complex potential shown in equation (5) is transformed by a conformal mapping function, the circulation Γ and the lift L' for the circular cylinder in the z- plane are the same as the circulation Γ and the lift L' in the w-plane, and this is, as a result of the angle and orientation preservation property of the conformal mapping .

In this study, we considered flow around an airfoil as an ideal fluid flow such that the flow is inviscid, meaning that have no viscosity, and incompressible, meaning that their density remains constant, and also satisfy the Kutta condition, which states that the fluid flowing over the upper and lower surfaces of the airfoil meets at the trailing edge of the airfoil, (Anderson, 2010) . Also the airfoil must be moving through the fluid at subsonic speeds. This is important because at speeds approaching the speed of sound**,** where compressibility effects in the fluid flow can be considered negligible, (Katz and Allen, 1991).

II. METHODOLOGY

The initial step to analyse lifting flow around airfoil, is to solve the flow around circular cylinder then relate the solution to airfoil by means of Joukowsky transformation equation (2)

Fluid Flow Around a Circular Cylinder

The lifting flow over a cylinder is a combination of a uniform flow, source-sink flow, doublet flow and vortex flow as follows:

Uniform Flow

Consider an ideal flow with a free stream uniform velocity V_∞ in the uniform positive x direction as shown in fig (1a). This flow can be defined by

$$\phi = V_\infty r\cos\theta \quad (8)$$

As potential function and

$$\psi = V_\infty r\sin\theta \quad (9)$$

As stream function

Obviously, (8) and (9) satisfies Laplace's equation, therefore the equipotential curves and streamlines are orthogonal.

Source-Sink Flow

If a flow is diverting from central point 0, the flow is referred as source flow, while if a flow is converging to the central point 0, the flow referred to as sink flow. In the source and sink flows all the streamlines are straight lines converging or diverging from central point 0, as shown in fig(1b).The resulting velocity field for these flows only has a radial component V_r, while is

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inversely proportional to the distance from 0. With these boundary condition in place, the potential and stream function for source and sink are given as

$$\phi = \frac{\Lambda}{2\pi} \ln r \qquad (10)$$

and

$$\psi = \frac{\Lambda}{2\pi}\, \theta \qquad (11)$$

respectively, where Λ is the rate of volume from source (source streangth) and r is the distance from 0. Note that, positive Λ value refers to source and negative Λ refers to sink.

Consider a uniform stream with the free stream velocity V∞ oriented from left to right. Superimpose it to a source flow of strength Λ located at the origin in polar coordinates as shown in fig (1b). The stream function for the resulting flow is simply found by addition of the stream functions of the two flows. It means

$$\psi = V_{\infty} r \sin\theta + \frac{\Lambda}{2\pi}\, \theta \qquad (12)$$

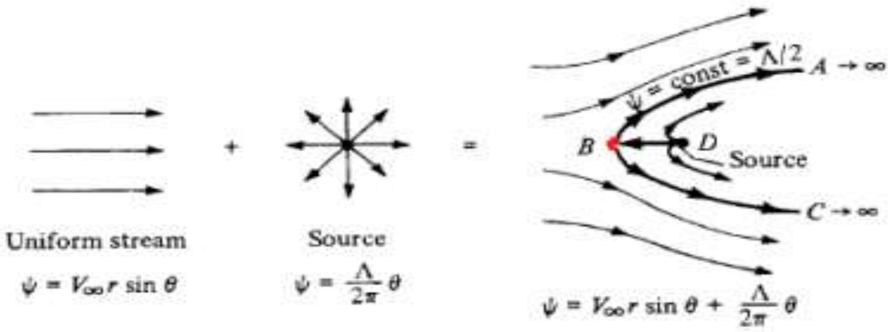

Figure (1a) Figure (1b) Figure (1c)

Figure (1): A combination (superposition) of a uniform flow and a source flow generates a flow over a semi-infinite body. Source : Anderson (2010).

Doublet Flow

A doublet flow is a particular degenerate case of source –sink combination that leads to a singularity,(Nyadwi , 2018). The potential function for doublet flow is

$$\phi = \frac{k}{2\pi} \frac{\cos\theta}{r} \qquad (13)$$

and stream function

$$\psi = -\frac{k}{2\pi} \frac{\sin\theta}{r} \qquad (14)$$

where k is the doublet strength. The superposition of the uniform and doublet flows provides a model of the non lifting flow around a cylinder. Adding potential functions given by (8) and (13)

$$\phi = V_{\infty} r \cos\theta + \frac{k}{2\pi} \frac{\cos\theta}{r} \qquad (15)$$

Equation (15) is justified by the linear nature of Laplace's equation. Note that setting $\psi = 0$ yields a circle of radius R given by

$$R = \sqrt{\frac{k}{2\pi V_{\infty}}} \qquad (16)$$

The resulting flow external to R is a valid model of the ideal flow around a cylinder. However, the entire flow field is symmetrical about the horizontal axis, meaning that this flow generates no lift on the cylinder. As shown in Fig 2

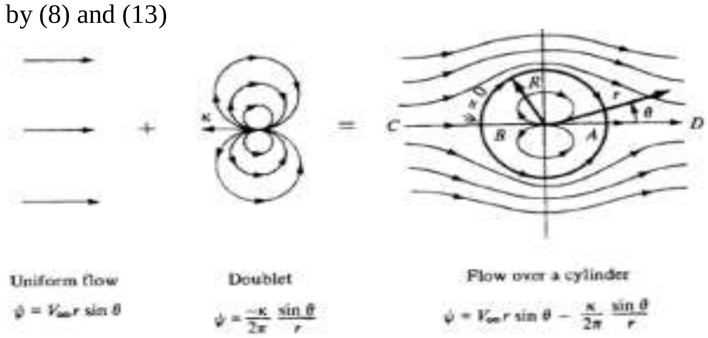

Figure (2a) Figure (2b) Figure (2c)

Figure 2: a combination (superposition) of a uniform flow and a doublet flow generates a non lifting flow over a circular cylinder.

Source: Chattot & Hafez, (2015).

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The superposition of the uniform flow and the doublet flow yields a non lifting flow over a circular cylinder. To model a lifting flow a vortex flow must be superimposed.

Vortex flow

A flow where all streamlines are concentric circles about a given point 0 is refers to as vortex flow. The tangential velocity V_θ is inversely proportional to the distance from 0. The resulting velocity potential of the vortex flow is given by

$$\phi = -\frac{\Gamma\theta}{2\pi} \qquad (17)$$

Where Γ is the circulation, that is the strength of the vortex.

The superposition of the uniform, doublet and vortex flows yields a potential function

$$\phi = V_\infty r\cos\theta - \frac{\Gamma\theta}{2\pi} + \frac{k}{2\pi}\frac{\cos\theta}{r} \qquad (18)$$

and stream function

$$\psi = V_\infty r\sin\theta + \frac{\Gamma}{2\pi}\ln r - \frac{k}{2\pi}\frac{\sin\theta}{r} \qquad (19)$$

The lifting flow over a cylinder is a combination of a non-lifting flow discussed above and a vortex flow of strength Γ as shown in the figure (3).

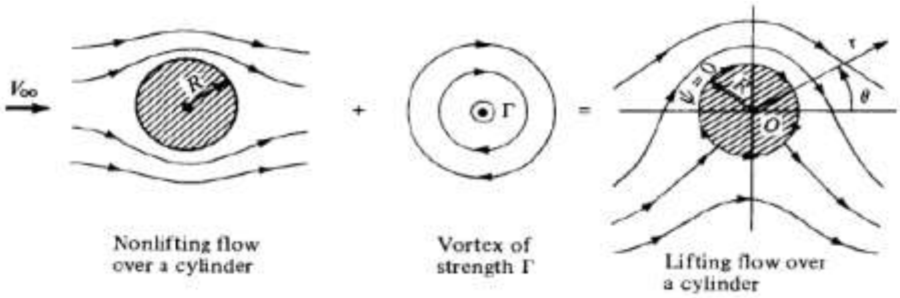

Figure (3a) Figure (3b) Figure (3c)

Figure 3: The synthesis flow of a lifting flow over a cylinder.

Source: Chattot & Hafez (2015).

The streamlines of this final superposition of three flows is shown in fig (4). As a result of vortex flow, the cylinder is now rotating with a finite angular velocity. This rotation eliminates the symmetry along the horizontal axis , creating an uneven pressure distribution, which generates lift. Equation (20) and (21) therefore are solutions to the lifting flow around a circular cylinder.

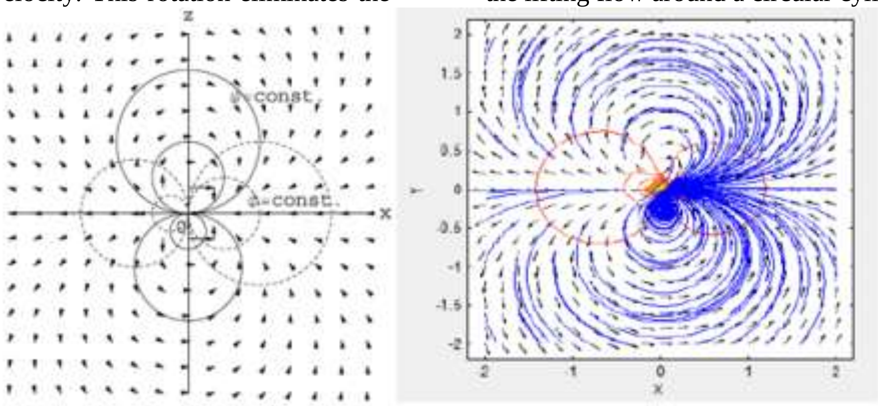

Figure 4: (Left) Doublet flow with strength k .Computed Doublet flow (right) in MATLAB with strength κ = 0.05 and streamlines (blue) and equipotential curves (red).

Source : Panton (2013).

Lift Around a Circular Cylinder

Consider an incompressible flow over an airfoil and let A be any curve in the fluid flow enclosing the airfoil, then the circulation is given by

$$\Gamma \equiv \oint_A V \cdot ds \qquad (20)$$

where V is the velocity field around the airfoil and the airfoil is generating a lift. It will turn out that the drag force is always zero; $F_d = 0$, and that the lift force is directly proportional to the

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circulation constant Γ. The exact relation for the lift force is

$L' = \rho V_\infty \Gamma$ (21)

Where ρ is the fluid density and V_∞ is the fluid velocity far upstream of the airfoil and Γ is the circulation defined in equation (20)

Joukowsky Mapping from Circular Cylinder to Airfoil

Having solved for the flow around a cylinder with the superposition of three elementary flows, we need to relate this solution to an airfoil shape. To accomplish this, we use a conformal mapping function called the Joukowsky transformation given by equation (2)

Consider,

$z = x + iy = r(\cos\theta + i\sin\theta) = re^{i\theta}$ (22)

$re^{i\theta}$ is a circle of radius r and the center at the origin in the z- plane (Lysak, 2011).

Therefore, the Joukowsky transformation, equation (2) in polar form gives

$$w = re^{i\theta} + \frac{\lambda^2}{r}e^{-i\theta}$$

(23)

$$\Leftrightarrow$$

$$w = (r\cos\theta + ir\sin\theta) + \frac{\lambda^2}{r}(\cos\theta - i\sin\theta)$$

$$= r\cos\theta + \frac{\lambda^2}{r}\cos\theta + r\,i\sin\theta - i\frac{\lambda^2}{r}(\sin\theta)$$

$$\Leftrightarrow$$

$$w = \left(r + \frac{\lambda^2}{r}\right)\cos\theta + i\left(r - \frac{\lambda^2}{r}\right)\sin\theta$$

(24)

Let

$a = r + \frac{\lambda^2}{r}$ and $b = r - \frac{\lambda^2}{r}$

Therefore,

$w = a\cos\theta + ib\sin\theta$ (25)

From

$$w = u + iv$$

Implies that.

$$u = a\cos\theta \Rightarrow \cos\theta = \frac{u}{a}$$

and

$$v = b\sin\theta \Rightarrow \sin\theta = \frac{v}{b}$$

From $\cos^2\theta + \sin^2\theta = 1$

Implies that,

$\left(\frac{u}{a}\right)^2 + \left(\frac{v}{a}\right)^2 = 1$ (26)

$$\Leftrightarrow$$

$\frac{1}{a^2}u^2 + \frac{1}{b^2}v^2 = 1$ (27)

If we set the parameter $\lambda = r$, then $a = r + \frac{r^2}{r}$ and $b = r - \frac{r^2}{r}$, and this implies that $a = 2r$ and $b = 0$,

and from this it can be seen that the circle in the z-plane is transformed into a flat plate of length $4r$ in the $w-$plane. It means that the points of the circle in z-plane occupy the strip $-2r \leq u \leq 2r$, in the w-plane, shown in fig (5).

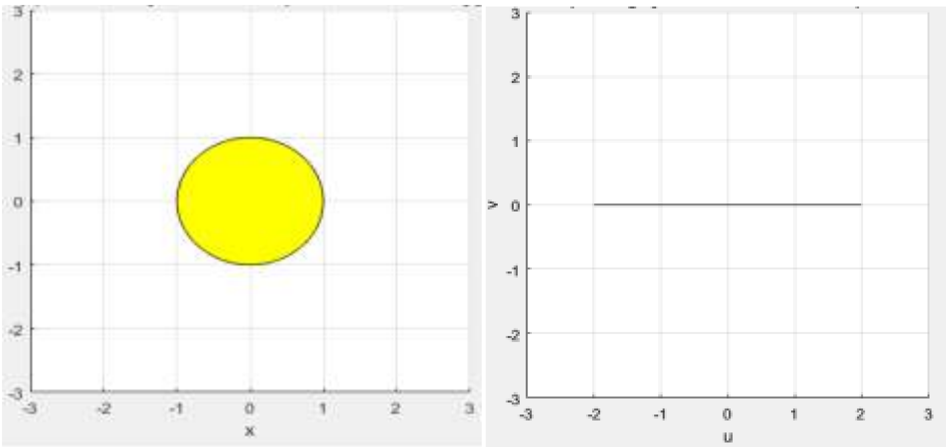

Figure 5: A unit circle in the z plane with center at the origin and corresponding flat plate in the w plane transformed using Joukowsky transformation, for $\lambda = 1$.

For $\lambda > r$,

$$\left(\frac{u}{r+\frac{\lambda^2}{r}}\right)^2 + \left(\frac{u}{r-\frac{\lambda^2}{r}}\right)^2 = 1 \quad (28)$$

$$\Leftrightarrow$$

$$\frac{u^2}{r^2(1+\lambda^2/r^2)^2} + \frac{v^2}{r^2(1-\lambda^2/r^2)^2} = 1 \quad (29)$$

Equation (29) is an equation of ellipse center at origin with major axis $a = r(1 + \lambda^2/r^2)$ and minor axis $b = r(1 - \lambda^2/r^2)$.Therefore the circle in z- plane is transformed into an ellipse in the w-plane. Shown in fig (6).

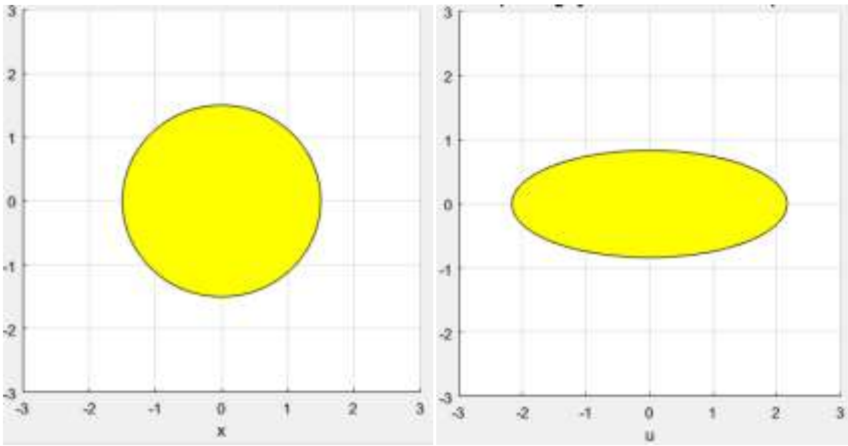

Figure 6: A unit circle in the z plane with center at the origin and corresponding ellipse in the w plane transformed using Joukowsky transformation, for $\lambda > 1$

However, neither of the shapes in fig.5 and 6 resemble an airfoil. The airfoil shape is realized by creating a circle in the z plane with a center that is offset from the origin, as shown in fig. 7 and 8. If the circle in the z plane is offset slightly, the desired transformation parameter is given as

$$\lambda = r - |t| \quad (30)$$

Where r is radius of a circle and t is the coordinates of the center of the circle (Panton, 2013).

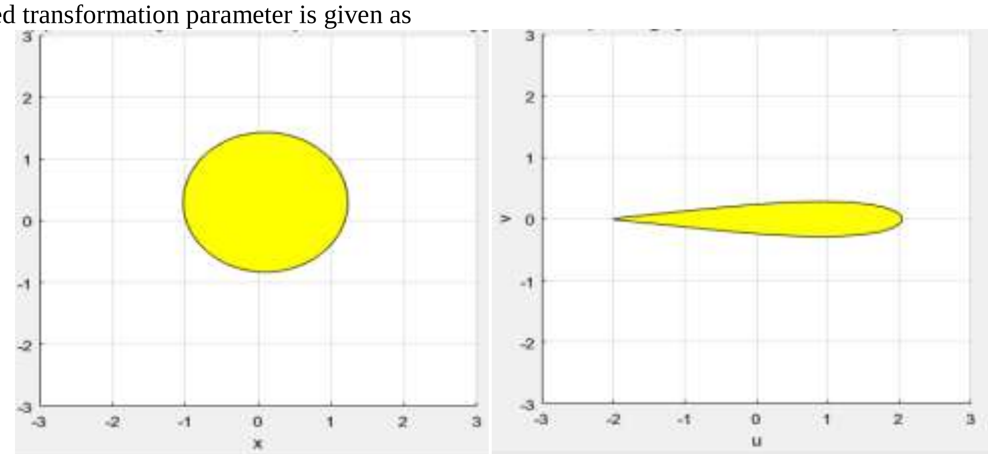

Figure 7: Cylinder in z plane with center offset on the x axis and the corresponding uncambered Joukowsky airfoil in the w plane with $\lambda = r - |t|$

The transformation in the w plane resembles the shape of an uncambered airfoil symmetric about the x axis, as shown in Fig. (7). The x coordinate of the circle origin therefore determines the thickness distribution of the transformed airfoil.If the center of the circle in the z plane is also offset on the y axis, the Joukowsky transformation yields an cambered airfoil as shown in Fig. (8). This shows that the y coordinate of the circle center determines the curvature of the transformed airfoil.

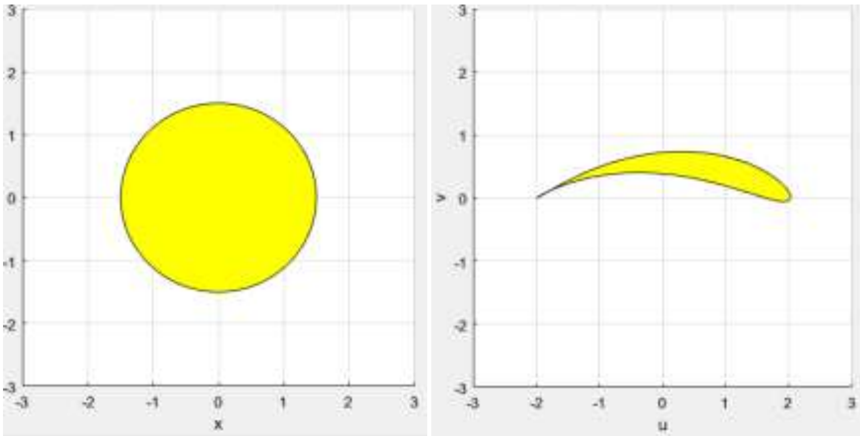

Figure 8: Cylinder in z plane with center offset on both x and y axis and the corresponding cambered Joukowsky airfoil in the w plane with $\lambda = r - |t|$

The airfoil shapes created from the Joukowsky transformation are known as Joukowsky airfoils. The x intercepts of the circle in the z plane become the leading and trailing edges of the mapped airfoil in the w plane, (Chattot & Hafez, 2015).

Note that, the generalised method for computation of the lift force on arbitrary airfoils cannot be executed manually; consequently, the method is coded in MATLAB for both symmetric and cambered airfoils (Swem, 2017).

III. RESULT

Having the solution for lifting flow around a cylinder and the technique for mapping this solution to the solution around a joukowsky airfoil, computational graphing program can be used to visualize the flow and establish lift calculation for several airfoils.

The contour plot of the imaginary component of the complex potential of the equation (5) gives the flow around the airfoil. The lift force is calculated using the formula in equation (21) where Γ for uncambered airfoil is given by;

$$\Gamma = \frac{2rV_\infty \sin\alpha}{2\pi} \tag{31}$$

From the MATLAB codes used, our result is presented by the following figures:

Flow Around a Circular Cylinder for $\propto = 0$. Lift Force: 0 N/M   Flow Around a corresponding Airfoil for $\propto = 0$. Lift Force: 0N/M

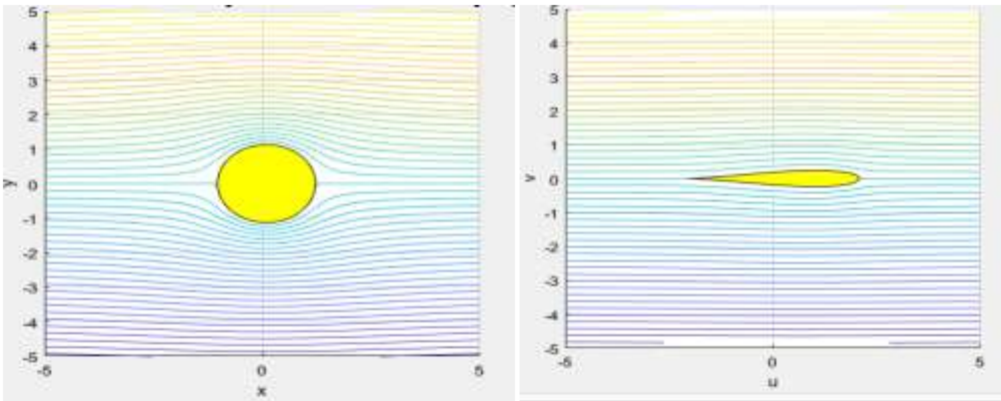

Figure 9: The streamlines around circular cylinder plot computed in the z plane and the corresponding uncambered Joukowsky airfoil. The plot was generated with $V_\infty = 200m/s$, $\alpha = 0$, and $\rho = 1.225\ kg/m^3$. The cylinder parameters used: $x = 0.1m$, $y = 0m$ and $r = 1.13m$

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Flow Around a Circular Cylinder for $\propto = 3$. Lift Force: 4.612 N/M Flow Around a corresponding Airfoil for $\propto = 3$. Lift Force: 4.612 N/M

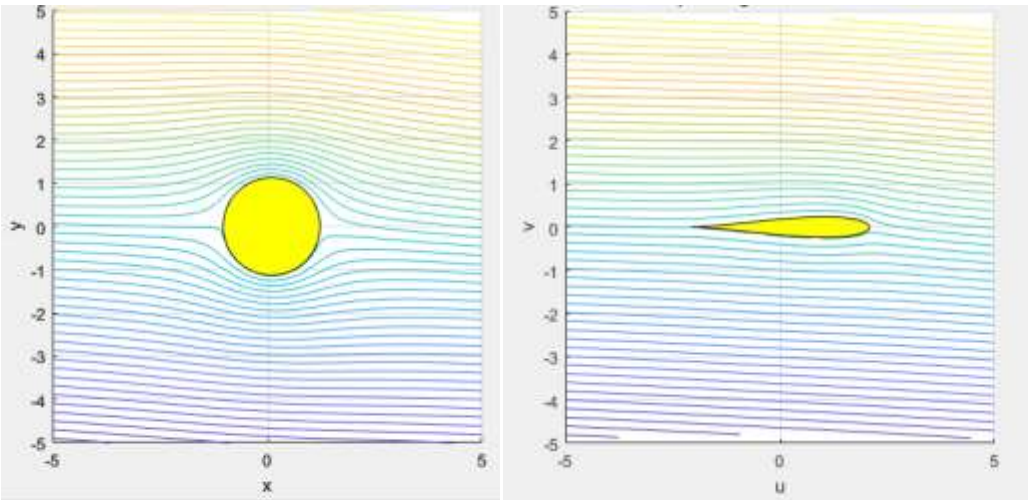

Figure 10: The streamlines around circular cylinder plot computed in the z plane and the corresponding uncambered Joukowsky airfoil. The plot was generated with $V_\infty = 200m/s$, $\alpha = 3$, and $\rho = 1.225\ kg/m^3$. The cylinder parameters used: $x = 0.1m,\ y = 0m, r = 1.13m.$

Flow Around a Circular Cylinder for $\propto = 5$. Lift Force: 7.6805 N/M Flow Around a corresponding Airfoil for $\propto = 5$. Lift Force: 7.6805 N/M

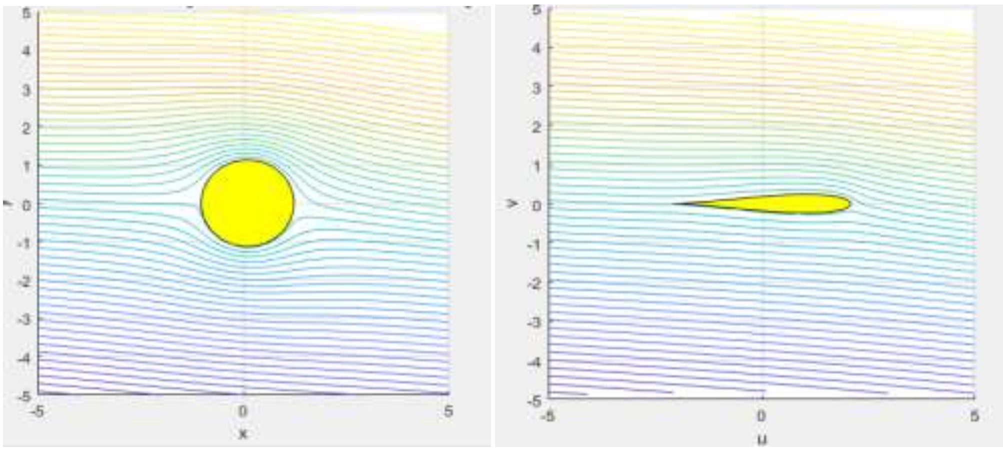

Figure 11: The streamlines around circular cylinder plot computed in the z plane and the corresponding uncambered Joukowsky airfoil. The plot was generated with $V_\infty = 200m/s$, $\alpha = 5$, and $\rho = 1.225\ kg/m^3$. The cylinder parameters used: $x = 0.1m,\ y = 0m, r = 1.13m.$

Flow Around a Circular Cylinder for Airfoil for $\propto = 7$. Lift Force: 10.7396 N/M Flow Around a corresponding for Airfoil for $\propto = 7$. Lift Force: 10.7396 N/M

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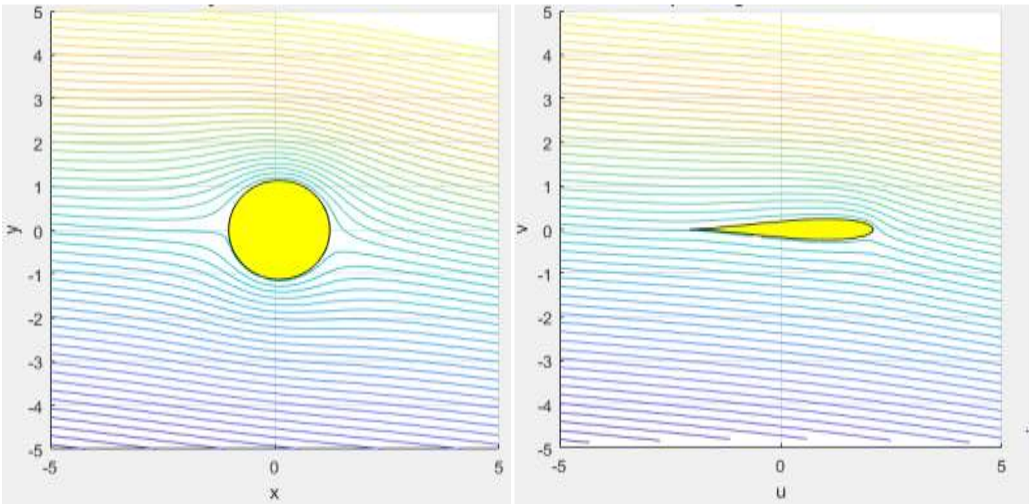

Figure 12: The streamlines around circular cylinder plot computed in the z plane and the corresponding uncambered Joukowsky airfoil. The plot was generated with $V_\infty = 200m/s$, $\alpha = 7$, and $\rho = 1.225\ kg/m^3$. The cylinder parameters used: $x = 0.1m,\ y = 0m, r = 1.13m$.

Table 1: Summary of results for lift force calculation

Angle of attack α	0	3	5	7
Lift force	0	4.6121	7.6805	10.7396

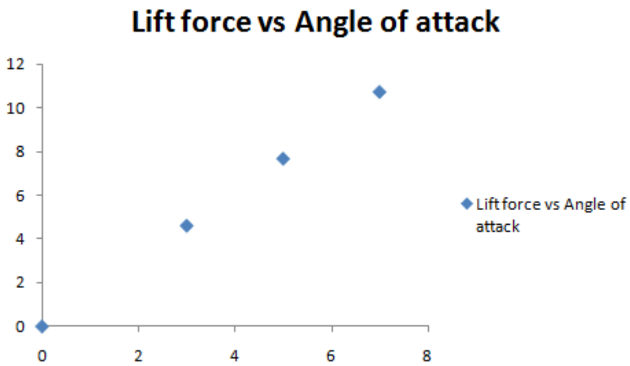

Figure 13: Lift force vs Angle of attack

IV. RESULT DISCUSSION

When a Joukowsky transformation is applied to an offset circular cylinder, one can get the Joukowsky airfoils by the use of instance of Matlab program. The streamlines generated by the imaginary part of the complex potential of the equation (5) is the flow solution around cylinder and corresponding symmetric airfoil. The lift force is calculated using the formula in equation (21) and the angle α that we have in the equation (31) was measured in radians and when converted into degrees amounts to $\frac{\alpha\pi}{180}$. In figure (9), at zero angle of attack, there is no lift generated on the cylinder and on the airfoil because the fluid flow is symmetric. This is due to the fact that there is a symmetric distribution of the streamlines about the x axis on both the circular cylinder in the z plane and the airfoil in the w plane. Again, looking at the airfoil, it is clear that the streamlines meet at the trailing edge, and therefore the Kutta condition is satisfied. As shown on table 1 above, we computed the streamlines around the circular cylinder and the corresponding airfoil at different values of angle of attack. In figure (10), the calculated lift force is 4.6121 N/m at α = 3, in figure (11), the lift force found is 7.6805 N/m at α = 5, and finally we have got 10.7396 N/m at α = 7, in figure (12).

The lift force was calculated at different angles of attack and it was found that the lift force is strongly dependent to the angle of attack in a linear proportion as shown in figure 13,

REFERENCES

[1]. Anderson D. J. (2010). Fundamentals of Aerodynamics, New York: Mc GrawHill.
[2]. Bear J., (1972). Dynamics of Fluids in Porous Media. Canada: American Elsevier,.
[3]. Benson S. &Thomas J., (1996). Interactive Educational Tool for Classical Airfoil Theory. Cleveland, Ohio: NASA Lewis Research Center, Oct. http://www.grc.nasa.gov/WWW/K-12/airplane/FoilTheory.pdf
[4]. Brown J. W. & Churchill R.V. (2009). Complex Variables and Application, McGraw-Hill Higher Education 8th ed, ISBN 978–0–07–305194–9 MHID 0–07–305194–2
[5]. Chattot J. J. & Hafez M . M., (2015), Theoretical and Applied Aerodynamics and Related Numerical Methods, Springer, New York.
[6]. Kapania N.R, Terracciano S. & Taylor S. (2008), Modeling the Fluid Flow around Airfoils Using Conformal Mapping, Society for Industrial and Applied Mathematics Undergraduate Research Online, Vol. 1, No. 2, pp. 70-99.
[7]. Katz J. & Allen P., (1991). Low Speed Aerodynamics. 2nd Edition. New York: McGraw-Hill.
[8]. Kazuo Y., Kazunari W. & Katsuyuki S., (2021), Generating various Airfoil shapes with required Lift coefficient using Condional Variational Auto Encoders, arXiv:2106.0991v1[cs. CS]
[9]. Lysak P. D., (2011), Unsteady Lift of Thick Airfoils in Incompressible Turbulent Flow, a PhD. Dissertation in Acoustics , Pennsylvania State University.
[10]. Maloneka H.R. & De Almeida R., (2010), A note on a generalized Joukowski transformation, Applied Mathematics Letters : 23 :1174–1178 : www.elsevier.com/locate/aml
[11]. Nyadwi B., (2018), Application of conformal Mapping to the Fluid flow, Msc Dissertation, Depaetment of Mathematics, University of Ruwanda.
[12]. Panton R. L., [Ed.] (2013), Incompressible Flow, New Jersey.
[13]. Prachi S., (2020), Conformal Transformation for airfoils, DOI:https://www.researchgate.net/publication/354464271
[14]. Sa Pai M. , (2020) , Applications of Kutta-Joukowski Transformationto the Flow Past a FlatPlate ,Vol. 4, Issue 6, ISSN: 2617-4537, pp 29-39
[15]. Swem S. T., (2017), Computation of Pressure Distribution and Aerodynamic Lift on Arbitrary Airfoils using Conformal Mapping, PhD Dissertation, Mathematical Sciences, Abubakar Tafawa Balewa University. Bauchi .
[16]. Swem S . T & Otene E., (2020). On the use of the s Schawrz- Chrittofel Transformation in the Solution and Analysis of Harmonic Dirichelet Problems of the Ideal Fluid Flows. Global Scientific Journals,vol.8, Issue 3,ISSN 2320-9186.