# ADOMIAN DECOMPOSITION METHOD FOR SOLVING HEAT TRANSFER ANALYSIS FOR SQUEEZING FLOW OF NANOFLUID IN PARALLEL DISKS.

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**Abstract:** Heat transfer analysis for the magneto-hydro-dynamics (MHD) Squeezing flow of a viscous incompressible fluid between parallel manifolds is considered. The upper manifold is movable in upward and downward directions while the lower manifold is fixed but permeable. Variable similarity transformation were used to convert the conservation law equations into a system of nonlinear ordinary differential equations. The resulting system of nonlinear ordinary differential equation were solved by using Adomian Decomposition Method (ADM). Influence of flow parameters is discussed and numerical solution is sought using Finite Difference Method (FDM). A convergent solution is obtained just after few number of iterations.

**Indexed Terms:** Squeezing flow, magneto-hydro-dynamics (MHD), Adomian Decomposition Method (ADM), Parallel manifolds, Finite Difference Method (FDM).

## 1.  INTRODUCTION

Heat transfer in rapidly moving engines and machines with lubricants inside has been an active field of research. For safe and consistent working of such machines it is necessary to study heat transfer in these systems. Several attempts are reported in this regard after the pioneer work done by Stefan [1]. Two dimensional **MHD** squeezing flow between parallel plates has been examined by Siddiqui et al. [2]. For parallel disk similar problem has been discussed by Domairy and Aziz [3]. Both used homotopy perturbation method **(HPM)** to determine the solution.

Joneidi et al. [4] studied the mass transfer effect on squeezing flow between parallel disks using homotopy analysis method **(HAM)**. Most recently, influence of heat transfer in **MHD** squeezing Flow between parallel disks has been investigated by Hayat et al. [5]. They used **HAM** to solve the resulting nonlinear system of ordinary differential equations.

Motivated by the preceding work here we present heat transfer analysis for the **MHD** squeezing flow between parallel disks. Well known Adomian Decomposition method **(ADM)** [6 -7] has been employed to solve system of highly nonlinear differential equations that govern the flow. **ADM** is a strong analytical technique and has been employed by several researchers in recent times to study different type of problems [8–9]. The main positive features of this technique is its simplicity, selection of initial approximation, compatibility with the nonlinearity of physical problems of diversified complex nature, minimal application of integral operator and rapid convergence [9]. We can also see that other methods have been employed by other researchers to tackle squeezing flow problem [10–26]

Numerical solution is also sought to check the validity of analytical solution. A detailed comparison between purely analytical solution obtained by ADM and the numerical solution obtained by employing FDM method is presented. It is evident from this article that the ADM provides excellent results with less amount of laborious computational work.

## 2.  Mathematical formulation

**MHD** flow of a viscous incompressible fluid is taken into consideration through a system consisting of two parallel infinite disks distance $h(t) = H(1 - at)^{1/2}$ apart. Magnetic field proportional to $B_0(1 - at)^{1/2}$ is applied normal to the disks. It is assumed that

there is no induced magnetic field. $T_W$ and $T_h$ represent the constant temperatures at $z = 0$ and $z = h(t)$ respectively. Upper disk at $z = h(t)$ is moving with velocity $\frac{aH(1-at)^{-1/2}}{2}$ toward or away from the static lower but permeable disk at $z = 0$ as shown in Fig. 1. We have chosen the cylindrical coordinates system $(r, \phi, z)$. Rotational symmetry of the flow $(\partial/\partial\phi = 0)$ allows us to take azimuthal component $v$ of the velocity $V = (u, v, w)$ equal to zero. As a result, the governing equation for unsteady two-dimensional flow and heat transfer of a viscous fluid can be written as [6]

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \qquad (1)$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial \hat{p}}{\partial r}$$
$$+ \mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial z^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right)$$
$$-\frac{\sigma}{\rho}B^2(t)u, \qquad (2)$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial \hat{p}}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial r^2} + \right.$$
$$\left. \frac{\partial^2 w}{\partial z^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right), \qquad (3)$$

$$C_P\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right)$$
$$= \frac{K_0}{\rho}\left(\frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2} + \frac{1}{r}\frac{\partial T}{\partial r} - \frac{u}{r^2}\right)$$
$$+ v\left\{2\frac{u^2}{r^2} + \left(\frac{\partial u}{\partial z}\right)^2 + \right.$$
$$2\left(\frac{\partial w}{\partial z}\right)^2 + 2\left(\frac{\partial u}{\partial r}\right)^2 + 2\left(\frac{\partial w}{\partial r}\right)^2 + 2\frac{\partial u}{\partial z}\frac{\partial w}{\partial r}\right\}, \qquad (4)$$

Auxiliary conditions are [5]
$$u = 0, \quad w = \frac{dh}{dt} \quad at \ z = h(t)$$
$$u = 0, \quad w = -w_0 \quad at \ z = 0.$$
$$T = T_w \quad at \ z = 0 \qquad (5)$$
$$u = T_h \ at \ z = h(t). \qquad (6)$$

$u$ and $w$ here are the velocity components in $r$ and $z$ directions respectively, $\mu$ is dynamic viscosity, $\hat{p}$ is the pressure and $\rho$ is the density. Further $T$ denotes temperature, $K_0$ is the thermal conductivity, $C_P$ is the specific heat, $v$ is the kinematic viscosity and $w_0$ is suction/injection velocity.

Using the following transformations [5]
$$u = \frac{ar}{2(1-at)}f'(\eta), \quad w = -\frac{aH}{\sqrt{1-at}}f'(\eta),$$
$$B(t) = \frac{B_0}{\sqrt{1-at}}, \quad \eta = \frac{aH}{\sqrt{1-at}}, \quad \theta = \frac{T - T_h}{T_w - T_h} \ (7)$$

Into Eqs. (2)-(4) and eliminating pressure terms from the resulting equations, we obtain

$$f^{iv} - S(\eta f''' + 3f'' - 2ff''') - M^2 f'' = 0, \qquad (8)$$
$$\theta'' - S\,Pr(2f\theta' - \eta\theta') - Pr\,Ec(f''^2 + 12\delta^2 f'^2) = 0 \qquad (9)$$

With the associated conditions
$$f(0) = A, \quad f'(0) = 0, \quad \theta(0) = 1,$$
$$\qquad (10)$$
$$f(1) = \frac{1}{2}, \quad f'(1) = 0, \quad \theta(1) = 0,$$

Where S denotes the squeeze number, $A$ is suction/injection parameter, $M$ is Hartman number, $Pr$ Prandtl number, $Ec$ modified Eckert number, and $\delta$ denotes the dimensionless length defined as
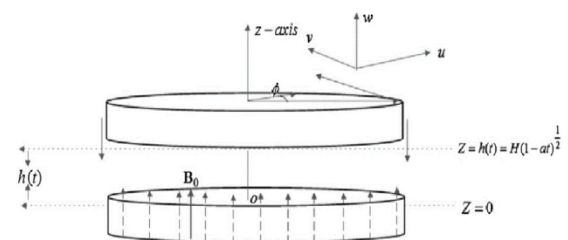
$$S = \frac{aH^2}{2v}, \quad M^2 = \frac{aB_0^2 H^2}{v}, \quad Pr = \frac{\mu C_p}{K_0},$$
$$Ec = \frac{1}{C_p(T_w - T_h)}\left(\frac{ar}{2(1-at)}\right)^2, \quad \delta^2 = \frac{H^2(1-at)}{r^2} \qquad (11)$$

Skin friction coefficient and the Nusselt number are defined in terms of variables (7) as

$$\frac{H^2}{r^2}Re_r C_{fr} = f''(1), \quad (1-at)^{1/2}Nu = -\theta'(1), \qquad (13)$$
$$Re_r = \frac{raH(1-at)^{1/2}}{2v}. \qquad (14)$$



## 3.    Adomian Decomposition Method (ADM)
The decomposition method was introduced by Adomian [3]. Consider the general equation:
$$\varphi[u(y)] = g(y) \qquad (15)$$
Where $\varphi$ represents a general non-linear ordinary (or partial) differential operator involving both linear and non-linear terms. The linear terms is decomposed

into the form $L + R$, where $L$ is usually taken as the highest order derivative which is assumed to be easily invertible and $R$ is the linear differential operator of order less than $L$. Therefore, equation (15) can be expressed as

$$Lu + Ru + Nu = g(y) \qquad (16)$$

Where $Nu$ represents the non-linear terms of $\varphi[u]$. Applying the inverse operator, $L^{-1}$ to both sides of equation (16) gives

$$u = L^{-1}g - L^{-1}(Ru + Nu). \qquad (17)$$

If $L$ is a fourth order operator, then $L^{-1}$ is a 4-fold integral. Now, solving equation (17), we have

$$u = \sum_{j=0}^{3} \alpha_j \frac{y}{j!} + L^{-1}g - L^{-1}(Ru + Nu), \qquad (18)$$

Where $\alpha_j (j = 1..3)$ are constants of integration and can be determined from the given boundary conditions.

The standard Adomian decomposition method defines the solution $u$ by the infinite series

$$u = \sum_{n=0}^{\infty} u_n \qquad (19)$$

And the non-linear term by the infinite series

$$Nu = \sum_{n=0}^{\infty} A_n \qquad (20)$$

Where $A_n$ are the Adomian polynomials determined formally from the relation;

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} N\left(\sum_{i=0}^{\infty} \lambda^i u_i\right)\right]_{\lambda=0}, \qquad i = 0,1,2,\dots \qquad (21)$$

The $u_n$ are determined from the recursive algorithm

$$u_0 = \sum_{j=0}^{3} \alpha_j \frac{y}{j!} + L^{-1}g$$

$$u_{n+1} = -L^{-1}(Ru + Nu) \quad n \geq 0, \qquad (22)$$

Where $u_0$ is the zeroth component. For numerical computation, the truncated series solution is obtained as

$$S_n(y) = \sum_{k=0}^{n-1} u_k \qquad (23)$$

Where $S_n$ denotes the $n$-term approximation of $u(y)$.

## 4.    ADM Solution Scheme

In this section, the solution to the system of non-linear differential equations $(8 - 9)$ subject to the boundary condition (10) is obtained via ADM.

The method is imperative because of the non-linearity involved. Equations $(8 - 9)$ can be written in operator form as:

$$L_1 f = S(\eta f''' + 3f'' - 2ff''') + M^2 f'' \qquad (24)$$

$$L_2 \theta = S \, Pr(2f\theta' - \eta\theta') + Pr \, Ec(f''^2 + 12\delta^2 f'^2) \qquad (25)$$

Where $L_1 = \frac{d^4}{d\eta^4}$ and $L_2 = \frac{d^2}{d\eta^2}$ is a fourth order and second order differential operator respectively, with inverse operators $L_1^{-1} = \int_0^\eta \int_0^\eta \int_0^\eta \int_0^\eta (\cdot) d\tau d\tau d\tau d\tau$ and $L_2^{-1} = \int_0^\eta \int_0^\eta (\cdot) d\tau d\tau$ respectively. Applying $L_1^{-1}$ to both sides of equation (24) and $L_2^{-1}$ to both sides of equation (25) and imposing the boundary conditions at $\eta = 0$ yields

$$f = A + \frac{\eta^2}{2}\alpha_1 + \frac{\eta^3}{6}\alpha_2 + L_1^{-1}[S(\eta f''' + 3f'' - 2N_1(f)) + M^2 f''] \qquad (26)$$

$$\theta = 1 + \eta\alpha_3 + L_2^{-1}[SPr(2N_2(f,\theta) - \eta\theta') + Pr \, Ec(N_3(f) + 12\delta^2 N_4(f))] \qquad (27)$$

Where $\alpha_1 = f''(0)$, $\alpha_2 = f''(0)$, $\alpha_3 = \theta'(0)$ are to be determined later using the boundary conditions at $\eta = 1$ and $N_1(f)$, $N_2(f,\theta)$, $N_3(f)$, $N_4(f)$ are the non-linear terms.

In terms of Adomian decomposition methods $f(\eta)$ and $\theta(\eta)$ are assumed to be a series solution of the form

$$f(\eta) = \sum_{n=0}^{\infty} f_n(\eta) \text{ and } \theta(\eta) = \sum_{n=0}^{\infty} \theta_n(\eta) \qquad (28)$$

And the non-linear terms are decomposed as series

$$N_1(f) = \sum_{n=0}^{\infty} A_n, \, N_2(f,\theta) = \sum_{n=0}^{\infty} B_n, \, N_3(f) = \sum_{n=0}^{\infty} C_n \text{ and } N_4(f) = \sum_{n=0}^{\infty} D_n \qquad (29)$$

Where $A_n$, $B_n$ and $C_n$ are the Adomian's polynomials which are generated by equation (21). Here

$$A_0 = f_0 f_0''', \, A_1 = f_0 f_1''' + f_1 f_0''', \dots,$$

$$B_0 = f_0 \theta_0', \, B_1 = f_0 \theta_1' + f_1 \theta_0', \dots,$$

$$C_0 = \left(f_0''\right)^2, \, C_1 = 2f_0'' f_2'' + \left(f_1''\right)^2, \dots,$$

$$D_0 = \left(f_0'\right)^2, \, D_1 = 2f_0' f_1', \dots,$$

Substituting equations (28-29) in equations (26-27) we obtain:

$$\sum_{n=0}^{\infty} f_n(\eta) = A + \frac{\eta^2}{2}\alpha_1 + \frac{\eta^3}{6}\alpha_2 + L_1^{-1}[S(\eta \sum_{n=0}^{\infty} f_n''' + 3\sum_{n=0}^{\infty} f_n'' - 2\sum_{n=0}^{\infty} A_n) + M^2 \sum_{n=0}^{\infty} f_n''], \qquad (30)$$

$$\sum_{n=0}^{\infty} \theta_n(\eta) = 1 + \eta\alpha_3 + L_2^{-1}[SPr(2\sum_{n=0}^{\infty} B_n - \eta\theta') + Pr \, Ec(\sum_{n=0}^{\infty} C_n + 12\delta^2 \sum_{n=0}^{\infty} D_n)] \qquad (31)$$

From integral equations (30-31), the recursive relations for the approximate analytical solution of system (8-10) are given as:

$$f_0 = A + \frac{\eta^2}{2}\alpha_1 + \frac{\eta^3}{6}\alpha_2 , \tag{32}$$

$$f_{n+1} = L_1^{-1}[S(\eta f''' + 3f'' - 2A_n) + M^2 f''] , \; n \geq 0, \tag{33}$$

$$\theta_0 = 1 + \eta\alpha_3 , \tag{34}$$

$$\theta_{n+1} = L_2^{-1}[SPr(2B_n - \eta\theta') + Pr\,Ec(C_n + 12\delta^2 D_n)] \; n \geq 0 \tag{35}$$

The following partial sum

$$f(\eta) = \sum_{n=0}^{\infty} f_k(\eta) \text{ and } \theta(\eta) = \sum_{n=0}^{\infty} \theta_k(\eta) \tag{36}$$

Are the approximate solutions. Equations (32-35) are coded using algebraic symbolic package called Maple.

## 5.    Results and Discussion

In this section, we present the graphical solution of the afore mentioned problem to show the effect of the pertinent parameters on the fluid flow. Which validates the ADM results given by the series solution (39), the 15th term ADM series solution is compared with those of numerical solutions obtained via Finite difference method (FDM) for fixed values of the parameters $S, A, \delta, Ec, Pr \; and \; M$, as shown in table 1. The table shows that the solution obtained from ADM are consistent and in good agreement with the FDM solutions.
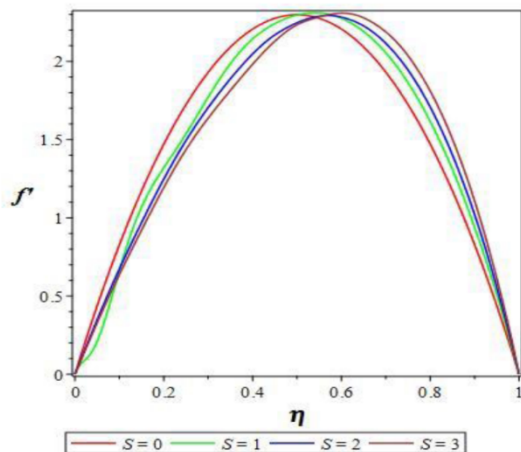


Figure 1: influence of squeezing parameter $S$ on $f'(\eta)$ when $M = 0.1, A = -1.0$ .
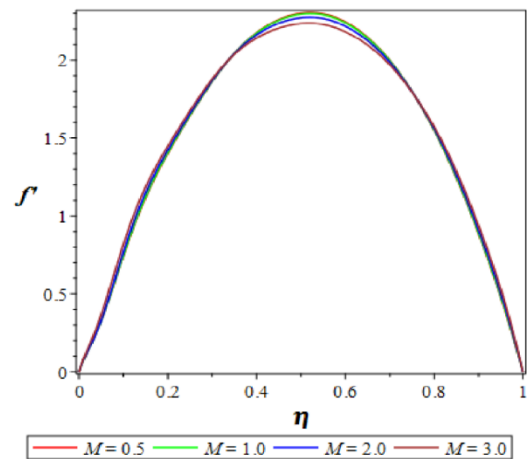


Figure 2: influence of Hartman number $(M)$ on $f'(\eta)$ when $S = 0.5, = -1.0$ .
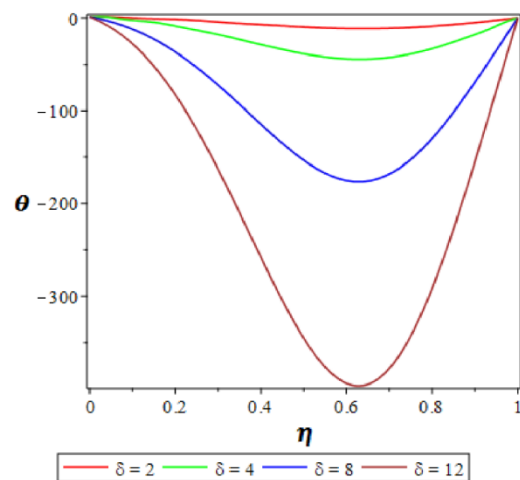


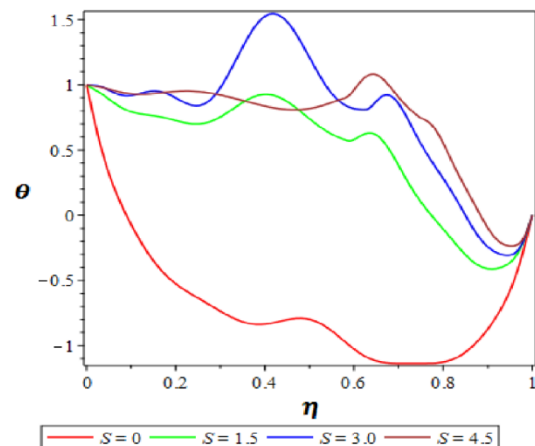Figure 3: influence of $(\delta)$ on $\theta(\eta)$ when $S = 0.5, Pr = Ec = 0.1. A = -1$



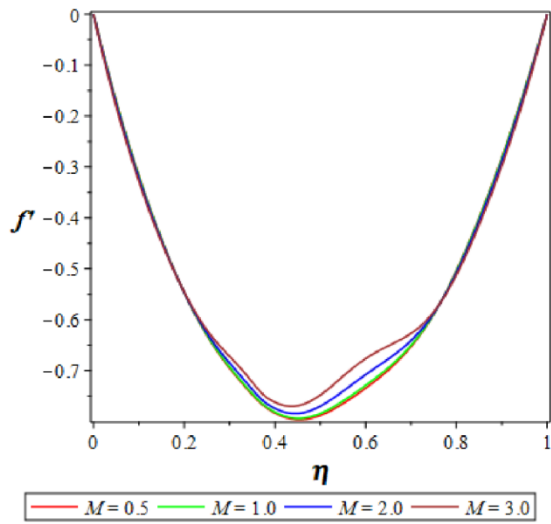Figure 4: influence of $S$ on $\theta(\eta)$ when $A = -1.0, Pr = Ec = 0.1, \delta = 0.10$ .

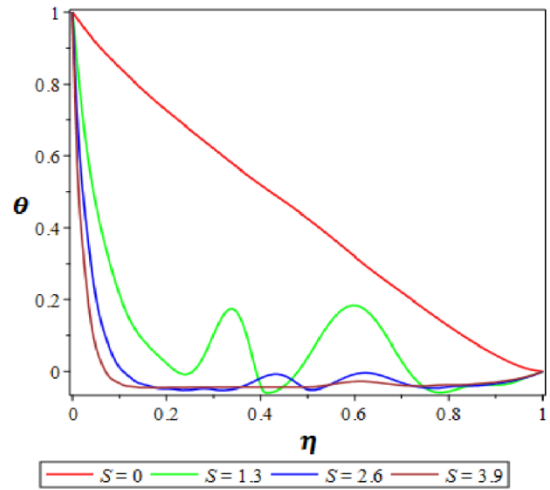Figure 5: influence of Hartman number $(M)$ on $f'(\eta)$ when $S = 0.5, A = 1.0$ .



Figure 7: influence of $S$ on $\theta(\eta)$ when $A = 1.0, Pr = Ec = 0.1, \quad \delta = 0.10$
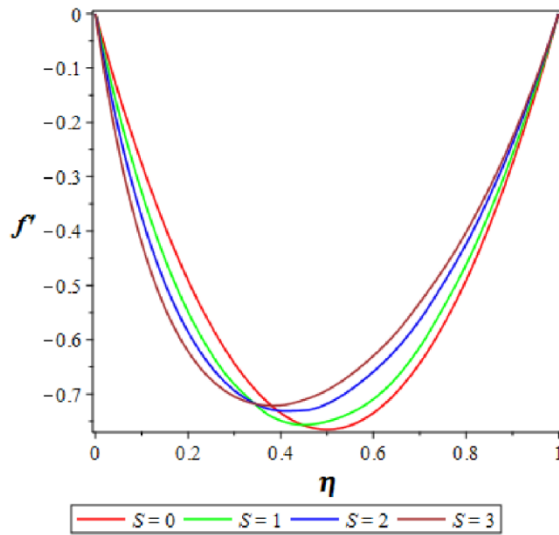


Figure 6: influence of squeezing parameter $S$ on $f'(\eta)$ when $M = 0.5, A = 1.0$ .
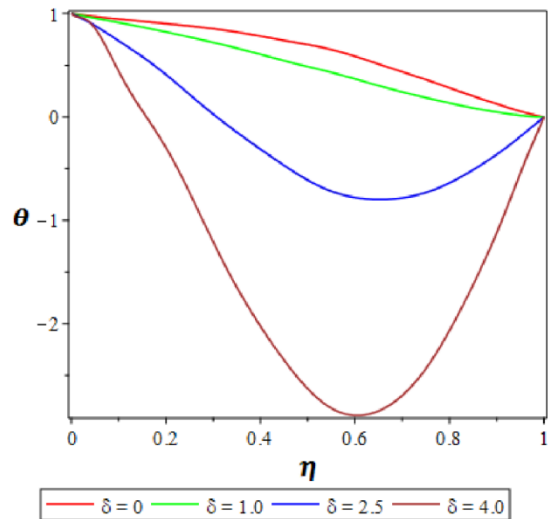


**Figure 8: influence of $(\delta)$ on $\theta(\eta)$ when $S = -0.5, Pr = Ec = 0.1. A = 1$**

Table 1.0 Comparison of Numerical and ADM solutions for diverging channel for $\delta = 0.1, A = 0.1, S = 0.1, M = 0.2, Pr = 0.3, Ec = 0.2$

| $\eta$ | $f'(\eta)$ | | | $\theta(\eta)$ | | |
|---|---|---|---|---|---|---|
| | FDM | ADM | Error | FDM | ADM | Error |
| 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0.2 | 0.384801 | 0.384801 | 0 | 0.793392 | 0.806144 | $1.3 \times 10^{-2}$ |
| 0.4 | 0.575554 | 0.575554 | 0 | 0.592274 | 0.607022 | $1.5 \times 10^{-2}$ |
| 0.6 | 0.575175 | 0.57517 | 0 | 0.392257 | 0.407004 | $1.5 \times 10^{-2}$ |
| 0.8 | 0.384040 | 0.384040 | 0 | 0.193382 | 0.206121 | $1.3 \times 10^{-2}$ |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |

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