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A fuzzy time series forecasting model based on Fuzzy C-means clustering technique and linear associations of input variables

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ABSTRACT: This study proposes a forecasting model using Fuzzy C-means (FCM) clustering technique combines with linear associations of input variables in fuzzy time series (FTS). The proposed model uses FCM to partition vectors of input data instead of clustering data of each variable separately and uses linear combinations of the input variables to establishes input/output relations instead of the fuzzy relations in conventional forecasting model. The cluster middles deal fuzziness and vagueness of the datum and the linear portions allow the algorithm to learn more from the existing information. The superior accuracy of the proposed model is demonstrated by experiments comparing it to other existing models using real-world empirical data
KEYWORDS: Fuzzy time series, linear regression, FCM, TAIEX, enrollments.

I. INTRODUCTION

Foundation of the theory of Fuzzy Time Series (FTS), which have been widely used in recent years, was laid by Song and Chissom [1, 2]. Values of fuzzy time series are fuzzy sets [3] and, there is a relationship between the observations at present time and those at previous times. There are certain features of fuzzy time series which makes it more suitable than the conventional forecasting systems. First of all, it can process both crisp and fuzzy values. In addition to that it does not require large datasets as in statistical forecasting models. The forecasting models developed by Song and Chissom [1,2] requires complex calculations due to max-min composition operations. Chen [4] proposed a more simplified model using arithmetic operations. This model is considered as the milestone in this particular field of research. It has been widely studied for improving accuracy of forecasting in many applications such as: university enrolments [1-13], crop production [14], stock markets [15-21], temperature prediction [20-21], etc. Particularly, Authors [18] presented a two-factor high-order FTS for the Taiwan futures exchange forecasting. It is noted that the forecasting accuracy rates of this model mainly depend on their universe of discourse and the length of intervals. Lee, Wang, and Chen [21] also developed a model for forecasting temperature and TAIFEX based on high-order FLR and genetic simulated annealing algorithm. By introducing genetic algorithm for partitioning intervals in the universe of discourse, Chen & Chung introduced the high - order [7] forecasting models for forecasting the enrolments of University of Alabama. Moreover, Particle swarm optimization algorithm has been successfully applied in many applications for the partition of the Universe of discourse. The comparative results show the forecasting models using Particle swarm optimization algorithm [11, 12, 19, 20] were generally found to perform better than other algorithms in terms of success rate and solution quality. Also, some comparative research works for the real problems presented that the PSO-based results have better performance than those based on GA [7, 21]. The above researches showed that the lengths of intervals and creating fuzzy logical relationships are two important issues considered to be serious influencing the forecasting accuracy and applied to different problems. However, most of the models were established based on fuzzy logical relationships for forecasting of other historical data.
The goal of the paper is to establish an efficient and precise model for forecasting FTS, based on fuzzy clustering and linear combinations of the input variables instead of the using fuzzy logical relationships.

II. PRELIMINARIES

In this section, we briefly review basic concepts of fuzzy time series [1, 2], the fuzzy C-means clustering algorithm [22] and the linear regression with least square estimation method [23].

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2.1. Some Basic Definitions of FTS

The idea of FTS was first introduced and defined by Song and Chissom [1-2]. Let $U = \{u_1, u_2, \dots, u_n\}$ be an universe of discourse; a fuzzy set A of U can be defined as $A = \{\mu_A(u_1)/u_1 +, \mu_A(u_2)/u_2 \dots + \mu_A(u_n)/u_n\}$,where μ_A: U→[0,1] is the membership function of A,$\mu_A(u_i)$indicates the degree of membership of u_i in the fuzzy set A, $f_A(u_i) \in [0, 1]$, and $1 \le i \le n$ .The basic definitions of FTS are as below:

Definition 1: Fuzzy time series [1, 2]

Let $Y(t)(t = .., 0, 1, 2 ..)$, a subset of real numbers, be the universe of discourse on which the fuzzy sets $f_i(t)$ $(i = 1,2 \dots)$ are defined in the universe of discourse Y(t) and F(t)is a collection of $f_1(t), f_2(t), \dots$, then F(t)is called a FTS definition on Y(t)(t . . ., 0, 1,2 . ..).

Definition 2: M-factors fuzzy time series[18]

Let O(t), H(t), L(t), C(t), F(t) be factors of the time series. For example, Open", \High", \Low", \Close", and \Final Price", respectively. If we only use F(t) to solve the forecasting problems, then it is called a one-factor time series. If we use remaining secondary-factors/secondary O(t), H(t), L(t), C(t) with F(t) to solve the forecasting problems, then it is called M-factors fuzzy time series.

Definition 3: Fuzzy logic relationship (FLR**)**[2-3]

The relationship between F(t) and F(t-1) can be denoted by $F(t-1) \rightarrow F(t)$. Let$A_j = F(t)$ and $A_i = F(t-1)$, the relationship between F(t) and F(t -1) is denoted by fuzzy logical relationship $A_i \rightarrow A_j$; where A_i and A_j refer to the current state or the left -hand side and the next state or the right-hand side of fuzzy time series.

2.2. Fuzzy C-means clustering algorithm [22]

Fuzzy C-Means is a method of clustering proposed by Bezdek. The basic idea of the fuzzy C-means clustering is described as follows: From a raw data set of input vectors $X = \{x_1, x_2, \dots, x_N\}$, the FCM employs fuzzy partitioning such that a data object can belong to two or more clusters with different membership grades between 0 and 1.The purpose of FCM is to minimize the following objective function:

$$J(U, V) = \sum_{i=1}^{C} \sum_{j=1}^{N} u_{ij}^{M} d_{ij}^{2}(x_j, v_i) \qquad (1)$$

Where, M is fuzziness parameter which is a weighting exponent on each fuzzy membership, C is the number of clusters $(2 \le C \le N)$, n is the number of objects in the data set X, v_i is the prototype of the center of cluster i, u_{ij} is the grade of membership of x_j belonging to cluster i and $d_{ij}^2(x_j, v_i)$ or d_{ij} is the distance between object x_j and cluster center v_i, U is the membership function matrix, V is the cluster center vector. The FCM focused on minimizing J (U, V), subject to the constrains (2) on U.

$$u_{ij} \in [0,1]; \quad \sum_{j=1}^{N} u_{ij} = 1; \quad \sum_{j=1}^{N} u_{ij} < N \qquad (2)$$

Algorithmic steps for Fuzzy C-means clustering is presented as follows:

Step 1: Fix the number of clusters C, initial the cluster center matrix V(0) by using a random generator from the original dataset. Record the cluster centers set t = 0, M = 2, and decided by ε , where ε is a small positive constant (i.e, $\varepsilon = 0.0001$).

Step 2: Initialize the membership matrix U(0) by using (3)

$$u_{ij}(t) = \frac{1}{\sum_{k=1}^{C} \left(\frac{d_{ij}(t)}{d_{kj}(t)} \right)^{\frac{2}{M-1}}} \qquad (3)$$

With $d_{ij} = \|x_j - v_i\|^2$ is is the distance between object x_j and cluster center v_i.

If $d_{ij}(t) = 0$, then $u_{ij} = 1$ and $u_{rj} = 0$ $(r \neq j)$.

Step 3: Increase t = t+1. Compute the new cluster center matrix V_{ij} using (4)

$$v_i(t+1) = \frac{\sum_{j=1}^{N} u_{ij}^{M}(t) * x_j}{\sum_{j=1}^{N} u_{ij}^{M}(t)} \qquad (4)$$

Step 4: Compute the new membership matrix U_{ij} by using (3)

Step 5: If $\max\{|u_{ij}(t+1) - u_{ij}(t)|\} < \varepsilon$ then stop, otherwise go to Step3 and continue to iterative optimization.

2.3. The linear regression with least square estimation approach

The linear regression with least square estimation method Linear Regression with respect to establishing linear relationships between dependent and independent variables. Such a relationship is portrayed in the form of an equation also known as the linear model. To solve this linear model, this study reviews the least square estimation method and shown as follows:

$$y_i = \beta_1 + \beta_2 x_{i,1} + \dots + \beta_p x_{i,p} + e_i \ (5)$$

Where, - y_i is the output variable of the ith data point

- $x_{i,1}$, $x_{i,2}$,.. and $x_{i,p}$ are the input variables of the ith data point, with p is the number of dimensions of the input variables, $(1 \le I \le n)$, n is the number of data points
- $\beta_1, \beta_2, \dots \beta_p$ are the coefficients that determine the relationship between the input variables and the output variable through an error constant e_i of the ith data point.

For n data points, the equation in (1) can be rewritten as follows:

$$Y = \beta X + E \qquad (6)$$

Where X is the input matrix and, Y is the output matrix, β is the coefficient matrix and E is the error matrix. The matrix is denoted as residuals that need to be minimized. These matrixes are calculated as follows:

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$$Y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix};\ X = \begin{pmatrix} 1 x_{1,1} \cdots x_{1,p} \\ 1 x_{2,1} \cdots x_{2,p} \\ \vdots\ \vdots\ \vdots\ \vdots \\ 1 x_{n,1} \cdots x_{n,p} \end{pmatrix};$$

$$\beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_p \end{pmatrix};\ e = \begin{pmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{pmatrix}$$

With the least square estimation method, the sum of squares of residuals will be minimized to estimate the matrix β . The estimation of the matrix β using the least square estimation method is shown as follows:

$$\beta = (X^T X)^{-1} X^T Y \quad (7)$$

III. A FORECASTING MODELCOMBINING FCM WITH LINEAR ASSOCIATIONS

In this section, a forecasting model based on Fuzzy C-means clustering algorithm and linear associations of input variables in fuzzy time series presented. The proposed model uses two FTSs F(t) and C(t). Where F(t) and C(t) are called the main-factor and the second-factor, respectively. In forecasting model using fuzzy logical relation, current state F(t-1) of the FLR of the forecast time t is constructed and then the fuzzy relation group with current state identical with that of the forecast FLR is found. Next state F(t) of the forecast FLR is taken the same as the next state of this FRG. Finally, based on the next state of FRG [4], crisp value of the forecast is calculated from defuzzification of the fuzzy value(s) of the forecast.

In the proposed model, the FCM algorithm is used to automatically generate clusters of historical training data and get the cluster centre of each cluster. Instead of FLRs, the proposed mode uses combinations of input variables to map the input data into the output space. For high- order FTS, one can simply apply FCM algorithm on the lagged variables of the FTS to forecast future values of the dependent variable. Based on the standard FCM algorithm in Section 2.2, the forecasting model combining FCM and FTS is presented as follows.

Consider $X_{r\times N}$ and $\vec{f}_{1\times N}$ as the input and output data of the FTS where N is number of observations and r is number of the factors of FTS. Jth input data vector is $\vec{x}_j = [\,x_{1j},\ x_{1j}, \ldots,\ x_{rj}]^T$ and its corresponding output is f_j. We group X matrix into k clusters using Fuzzy C-means algorithm [23]. For this purpose, following index is minimized with the given constraint.

$$J_1 = \sum_{j=1}^{N}\sum_{i=1}^{c} u_{ij}^m \left\|\vec{x}_j - \vec{v}_i\right\|_A^2, \sum_{i=1}^{c} u_{ij} - 1 = 0 \quad (8)$$

Where, c is the number of clusters, $\vec{v}_i$ is center of the i^{th} cluster (i^{th} row of cluster centers matrix, $V_{r\times c}$), u_{ij} is membership grade of the j^{th} data vector in the i^{th} cluster (element of partition matrix, $U_{c\times N}$), $\left\|\vec{x}_j - \vec{v}_i\right\|_A^2 = \left(\vec{x}_j - \vec{v}_i\right)^T A\left(\vec{x}_j - \vec{v}_i\right)$ is distance, $m \in (1,\infty]$ is degree of fuzziness and $A_{r\times r}$ is the covariance norm matrix. The values of $\vec{v}$ and u_{ij} are calculated as:

$$\vec{v}_i = \frac{\sum_{j=1}^{N} u_{ij}^m \vec{x}_j}{\sum_{j=1}^{N} u_{ij}^m}, u_{ij} = \left[\sum_{k=1}^{c}\left(\frac{\left\|\vec{x}_j - \vec{v}_i\right\|_A^2}{\left\|\vec{x}_j - \vec{v}_k\right\|_A^2}\right)^{\frac{1}{m-1}}\right]^{-1} \quad (9)$$

These equations are repeatedly updated until changes in U and Vbecome negligible. After determining on cluster centers, Membership Function (MF) of the q^{th} variable in the i^{th} cluster is calculated as:

$$u_{qij} = \left[\sum_{k=1}^{c}\left(\frac{\left\|x_{qj}-v_{qi}\right\|^2}{\left\|x_{qj}-v_{qk}\right\|^2}\right)^{\frac{1}{m-1}}\right]^{-1} \quad (10)$$

$$\left\|x_{qj} - v_{qi}\right\|^2 = \left(x_{qj} - v_{qi}\right)^2\ \forall j \in [1, N] \quad (11)$$

Weighted contribution of each cluster in the calculation of the output associated with $\vec{x}_j$ is calculated as:

$$\tau_{ij} = \frac{\prod_{q=1}^{r} u_{qij}}{\sum_{i=1}^{c}\prod_{q=1}^{r} u_{qij}} \quad (12)$$

Consider the matrix $X^*_{(r+1)\times N}$ such that: $x^*_{1j} = 1, x^*_{(q+1)j} = x_{qj}\ \forall q \in [1, r], j \in [1, N]$. Then output value F_j of model for $\vec{x}^*_j = [1\quad \vec{x}_j]$ is considered as the weighted linear combinations of the input variables.

$$F_j = \sum_{i=1}^{c} \tau_{ij} \sum_{q=1}^{r+1} p_{iq} x^*_{qj} \quad (13)$$

The coefficients p_{iq} are obtained by minimizing the following index:

$$J_2 = \left(\sum_{i=1}^{c} \tau_{ij} \sum_{q=1}^{r+1} p_{iq} x^*_{qj} - F_j\right)^2 \quad (14)$$

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Since number of equations in $H\vec{P}=\vec{F}$ is usually higher than number of unknowns, it is solved by Least Square Estimate method (LSE), where the error $e=\left(\vec{F}-H\vec{P}\right)^T\left(\vec{F}-H\vec{P}\right)$ is minimized.

Where, $(H^TH)^+$ is pseudo-inverse of H^TH; $\vec{P}$ and H are defined as follows:

$$\vec{P}=(H^TH)^+H^T\vec{F} \qquad (15)$$

Where;

$$H=\begin{bmatrix} \tau_{11} & .. & \tau_{c1} & \tau_{11}x_{11} & .. & \tau_{c1}x_{11} & .. & \tau_{11}x_{r1} & .. & \tau_{c1}x_{r1} \\ \tau_{12} & .. & \tau_{c2} & \tau_{12}x_{12} & .. & \tau_{c2}x_{12} & .. & \tau_{12}x_{r2} & .. & \tau_{c2}x_{r2} \\ \vdots & \cdots & \vdots & \vdots & .. & \vdots & .. & \vdots & .. & \vdots \\ \tau_{1j} & .. & \tau_{cj} & \tau_{1j}x_{1j} & .. & \tau_{cj}x_{1j} & .. & \tau_{1j}x_{rj} & .. & \tau_{cj}x_{rj} \\ \vdots & .. & \vdots & \vdots & .. & \vdots & .. & \vdots & .. & \vdots \\ \tau_{1N} & .. & \tau_{cN} & \tau_{1N}x_{1N} & .. & \tau_{cN}x_{1N} & .. & \tau_{1N}x_{rN} & .. & \tau_{cN}x_{rN} \end{bmatrix}$$

$$\vec{P}=[\vec{p}_1 \quad \vec{p}_2 \quad \cdots \quad \vec{p}_i \quad \cdots \quad \vec{p}_c]^T$$
$$\vec{p}_i=[p_{i1} \quad p_{i2} \quad \cdots \quad p_{iq} \quad \cdots \quad p_{i(r+1)}]$$

The forecasting performance of proposed model is assessed with help of the root mean square error (RMSE) to compare the difference between the forecasted values and the actual values. The RMSE is calculated according to formula (16) as follows

$$\text{RMSE}=\sqrt{\frac{1}{n}\sum_{j=1}^{n}\left(F_j-R_j\right)^2} \qquad (16)$$

Where, R_j denotes actual value, F_j is forecasted output value, n is number of the forecasted data used for training or testing.

IV. EXPERIMENTAL RESULTS

This study, we focus on two time series datasets which are often used to demonstrate validity and performance of the FTS forecasting model. The statistical characteristics of two these time series are expressed as follows.

4.1. Time series description

a) The enrolments dataset of university of Alabama

This time series data consists of 22 values between 1971 and 1992, see Fig .1. This dataset has utilized to examined with the huge amount of reseach works which are presented in the articles [1-13,25]. The obtained results among these works are choosed for comparing with our proposed model. Some of results among these studies are considered for comparing with that of the proposed model in this paper. Seven clusters are used in this study for comparing other forecasting model.

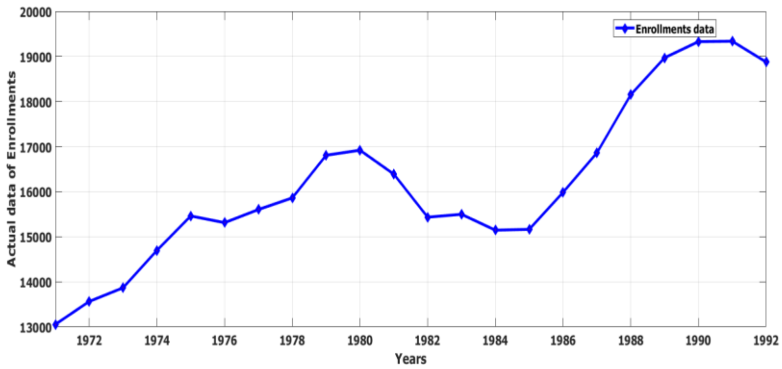

Fig. 1. Historical data of enrollments of the University of Alabama

b) TAIEX dataset

The dataset consists of daily values of Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) from 01/2014 to 12/2014, which consists of 248 observations. TAIEX data are frequently used for evaluation of the FTS forecasting model which is presented in the literatures [15-21]. This data has two variables, where 226 of them are used for training and the remaining data are used for testing. For all the TAIEX data sets studied in this work, the close price f is considered as a function of the high price r1 and low price r2. We use seven clusters as [19]. The following step-by-step procedure is used to apply CFTS algorithm on these data.

4.2. Applying for real-world problems

4.2.1. Forecasting enrolments of University of Alabama

In this section, the proposed forecasting model is handled for forecasting enrolments whose yearly observations [4] and shown in Fig.1.The forecasting results of the proposed model for training is shown in Table 1 and Fig. 2.

Table 1: The results of proposed model for forecasting enrollments

Year	Actual data	Forecasted value
1971	13055	
1972	13563	14825.13
1973	13867	15393.9
1974	14696	15388.6
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1990	19328	19305.8
1991	19337	19111.9
1992	18876	19111.8

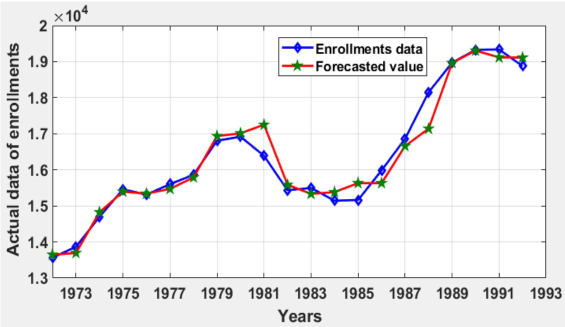

Fig. 2: The curves of the actual data and forecasted data for forecasting the enrolments of Alabama

To prove the performance of the proposed forecasting model based on the first order FTS under different number of intervals, four forecasting models presented in articles [4, 2, 17, 21, 24] which are selected for comparing.Table 2 shown a comparison of the RMSE value for different forecasting models.

Based on forecasted results in Table 2, the proposed model gets the smallest RMSE value of 288.27 among all the compared models with number of clusters equal to 7. This can be seen that the proposed model gives the most accurate forecasting results for enrolments of University of Alabama.

Table 2: The results of the proposed model and the compared models with 7 intervals

Models	RMSE
Chen [4]	638.36
Song and Chissom [2]	605.4
Sullivan and Woodall [25]	621.3
Huarng [17]	476
Cheng et al.,[24]	478.5
Wei Lu [21]	445.2
Proposed model	288.27

4.2.2. ForecastingforTAIEX2014

In this experiment, the TAIEX stock index, from 2/1/2014 to 31/12/2014, is selected to initially test the proposed model. The forecasting results of the proposed model is depicted in Fig.3.The forecasting results obtained from these experiments in training stage are compared with the ones of the current models [2-4,17, 25, 26] under the same

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number of intervals equal to 7. A comparison with regards to RMSE value between the proposed model and the different forecasting models are given in Table 3. Considering the Table 3, the results show that the proposed model has the smallest RMSE value equal to 23.73 among all its counterparts. Moreover, from Fig. 3, it can be seen that the forecasted value is close to the actual data on each day for the TAIEX data, from 2/1/2014 to 31/12/2014.

Table 3: The results of the proposed model and the compared models for forecasting TAIEX 2014

Models	RMSE
Song and Chissom [2]	90.03
Song and Chissom [3]	85.72
Chen [4]	84.44
Sullivan and Woodall [25]	82.79
Huarng [17]	78.27
Kuo et al.,[26]	60.25
Proposed model	23.73

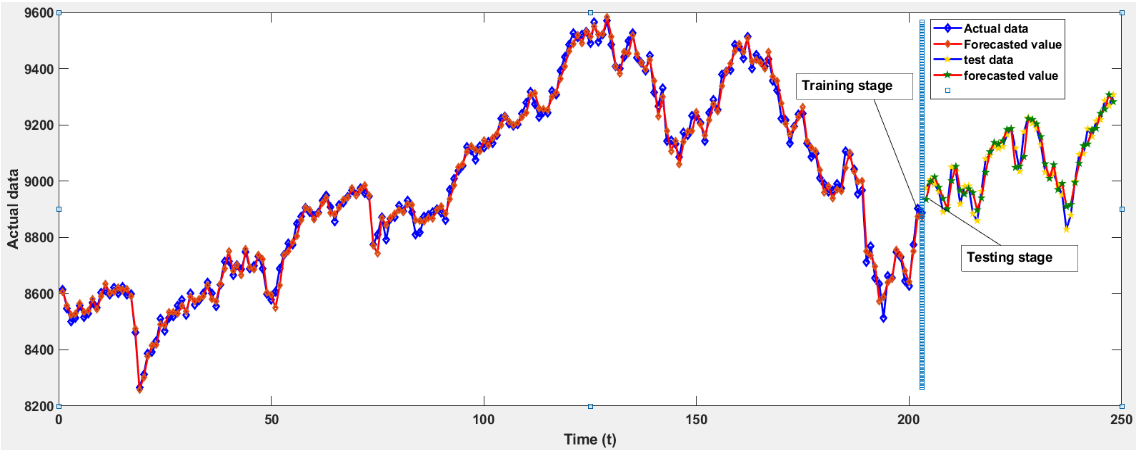

Fig.3: The curves of the actual data and forecasted data for the TAIEX data of 2014.

V. CONCLUSION

A clustering-based forecasting algorithm for FTS is proposed. A hybrid FTS model deals with ambiguity, vagueness and uncertainty of the fuzzy time series using fuzzy clusters and replaces fuzzy relations of the conventional FTS algorithms with linear combinations of the input variables. Parameters of the linear combinations are estimated by the Least Square Estimate which enable the algorithm to learn behavior of the data more precisely compared to the existing FTS algorithms. The forecasting model is evaluated using various datasets. The experimental results on two data sets of enrolments and TAIEX2014show that the proposed model outperforms other forecasting methods with number of interval equal to 7.

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