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A Generalized Sub-ODE Method

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ABSTRACT: In this paper, we derive exact traveling wave soluti-ons of (2+1) dimensional breaking soliton equation by a proposed Bernoulli sub-ODE method.The method appears to be efficient in seeking exact solutionsof nonlinear equations. We also make a comparison between the present method and the known (G'/G) expansion method.

KEYWORDS: Bernoulli sub-ODE method, traveling wave solutio-ns,exact solution, evolution equation, (2+1) d imensional KDV equation.

I. INTRODUCTION

The nonlinear phenomena exist in all the fields including either the scientific work or engineering fields, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics, and so on. It is well known that many non-linear evolution equations (NLEEs) are widely used to describe these complex phenomena. Research on solutions of NLEEs is popular. So, the powerful and efficient methods to find analytic solutions and numerical solutions of nonlinear equations have drawn a lot of interest by a diverse group of scientists. Many efficient methods have been presented so far.

In this paper, we pay attention to the analytical method for getting the exact solution of some NLEES. Among the possible exact solutions of NLEEs, certain solutions for special form may depend only on a single combination of variables such as traveling wave variables. In the literature, Also there is a wide variety of approaches to nonlinear problems for constructing traveling wave solutions. Some of these approaches are the inverse scattering transform, theDarboux transform, the tanh-function expansion and its various extension, the Jacobi ellipticfunction expansion, the homogeneous balance method, the sine-cosine method, the rank analysismethod, the exp-function expansion method and so on [1-5]. In this paper, we propose a Bernoulli sub-ODE method which is similar to the known (G'/G)-expansion method, and we will constructexact traveling wave solutions for NLEES by the proposed Bernoulli sub-ODE method.

First we recall the known the (G'/G)-expansion method. Suppose that a nonlinear equation, say in three independentvariables x, y and t , is given by

$$P(u,u_{t},u_x,u_y,u_{tt},u_{xt},u_{yt},u_{xx},u_{yy}......)=0$$

(1.1)

where u = u(x, y, t) is an unknown function,P isa polyno-mial in u = u(x, y, t) and its various partialderivatives, in which the highest order derivatives andnonlinear terms are involved. The (G'/G)-expansion method is applied in four steps.

Step 1. Combining the independent variables x, y and t

into one variable $\xi=\xi(x,y,t)$, we suppose that

$$u(x,y,t)=u(\xi),\xi=\xi(x,y,t) \quad (1.2)$$

the travelling wave variable (2.2) permits us reducing Eq.

(1.1) to an ODE for $u=u(\xi)$

$$P(u,u',u'',......)=0 \quad (1.3)$$

Step 2. Suppose that the solution of (1.3) can be expre-ssedby a polynomial in (G'/G) as follows:

$$u(\xi)=\alpha_m(\tfrac{G'}{G})^m+...... \quad (1.4)$$

where $G=G(\xi)$ satisfies the second order LODE in the

form

$$G''+\lambda G'+\mu G=0 \quad (1.5)$$

$\alpha_m,...\lambda$ and μ are constants to be determined later, $\alpha_m\neq 0$ .The unwritten part in (1.4) is also a polynomial in $(\frac{G'}{G})$, the degree of which is generally equal to or less than m−1.The positive integer m can be determined by consider-ing thehomogeneous balance between the highest order de-rivativesand nonlinear terms appearing in (1.3).

Step 3. Substituting (1.4) into (1.3) and using second order LODE (2.5), collecting all terms with the sameorder of $(\frac{G'}{G})$ together, the left-hand side of Eq. (1.3)is converted into another polynomial in

$(\frac{G'}{G})$. Equatingeach coefficient of this polynomial to zero, yields a set ofalgebraic equation-ns for $\alpha_m,...\lambda$ and μ.

Step 4. Assuming that the constants $\alpha_m,...\lambda$ and μ canbe obtained by solving the algebraic equations in Step 3, since the general solutions of the second order LODE (1.5) have been well known for us, substituting $\alpha_m,...$ and the general solutions of Eq. (1.5) into (1.4) we can obtain the traveling wave solutions of the nonlinear evolution equation (1.1).

The rest of the paper is organized as follows. In Section 2, we describethe Bernoulli sub-ODE method for finding traveling wave solutions of nonlinearevolution equations, and give the main steps of the method. In thesubsequent section, we will apply the Bernoulli Sub-ODE method to find exact traveling wave solutionsof the (2+1) dimensional KDV equation.In the last section, some conclusions are presented.

II. DESCRIPTION OF THE BERNOULLI SUB-ODE METHOD

In this section we present the solutions of the following ODE::

$$G'+\lambda G=\mu G^2 \qquad (2.1)$$

where $\lambda \neq 0, G=G(\xi)$

When $\mu \neq 0$, Eq.(2.1) is the type of Bernoulli equation, and we canobtain the solution as

$$G=\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}} \qquad (2.2)$$

where d is an arbitrary constant.

Suppose that a nonlinear equation, say in two or three independentvariables x, y and t , is given by

$$P(u,u_t,u_x,u_y,u_{tt},u_{xt},u_{yt},u_{xx},u_{yy}......)=0$$

(2.3)

where u = u(x, y, t) is an unknown function,P isa polyno-mial in u = u(x, y, t) and its various partialderivatives, in which the highest order derivatives andnonlinear terms are involved.By using the solutions of Eq.(2.1),we can constr-uct a serials of exact solutions of nonlinear equations:.

Step 1.We suppose that

$u(x,y,t)=u(\xi), \xi=\xi(x,y,t)$ (2.4) the travelling wave variable (2.4) permits us reducing Eq.(2.3) to an ODE for $u=u(\xi)$

$$P(u,u',u'',......)=0 \qquad (2.5)$$

Step 2. Suppose that the solution of (2.5) can be expre-ssedby a polynomial in G as follows:

$$u(\xi)=\alpha_m G^m+\alpha_{m-1}G^{m-1}+...... \quad (2.6)$$

where $G=G(\xi)$ satisfies Eq.(2.1), and $\alpha_m,\alpha_{m-1}...$ are constants to be determined later, $\alpha_m \neq 0$.The positive integer m can be determined by considering thehomogen-eous balance between the highest order derivativesand non-linear terms appearing in (2.5).

Step 3. Substituting (2.6) into (2.5) and using (2.1), collecting all terms with the sameorder of G together, the left-hand side of Eq. (2.5)is converted into another polynomial in G. Equatingeach coefficient of this polynomial to zero, yields a set ofalgebraic equation-ns for $\alpha_m,\alpha_{m-1},...\lambda,\mu$.

Step 4. Solving the algebraic equations system in Step 3, and by usingthe solutions of Eq.(2.1), we can construct the traveling wave solutions of thenonlinear evolution equation (2.5)..

In the subsequent sections we will illustrate the propo-sedmethod in detail by applying it to (2+1) dimensional breaking soliton equation.

since the general solutions of the second order LODE (2.5)have been well known for us, substituting $\alpha_m,...$ and thegeneral solutions of Eq. (2.5) into (2.4) we can obtain thetraveling wave solutions of the nonlinear evolution equation(2.1).

In the subsequent sections we will illustrate the propo-sedmethod in detail by applying it to a nonlinear evolutionequation.

III. APPLICATION OFTHE BERNOULLI SUB-ODE METHOD FOR(2+1) DIMENSIONAL KDV EQUATION

In this section, we will consider the following (2+1) dimensional breaking soliton equation:

$$u_t+u_{xxx}-3(uv)_x=0 \quad (3.1)$$

$$u_x=v_y \quad (3.2)$$

Suppose that

$$u(x,y,t)=u(\xi), v(x,y,t)=v(\xi)$$

$$\xi=k(x+y-ct) \qquad (3.3)$$

k,c are constants that to be determined later.

By (3.3), (3.1)-(3.2) are converted into the following ODEs

$-cu'+k^2u'''-3(uv)'=0$ (3.4)

$u'=v'$ (3.5)

Integrating (3.4)-(3.5) once we obtain

$-cu+k^2u''-3uv=g_1$ (3.6)

$u-v=g_2$ (3.7)

where g_1, g_2 are the integration constants.

Suppose that the solution of (3.6)-(3.7) can be expressed by apolynomial in G as follows:

$u(\xi)=\sum_{i=0}^{m} a_i G^i$ (3.8)

$v(\xi)=\sum_{i=0}^{n} b_i G^i$ (3.9)

Where a_i, b_i are constants, and $G=G(\xi)$ satisfies Eq.(2.1).

Balancing the order of u'' and uv in Eq. (3.6) and the order of u and v in Eq. (3.7), we have

$m+2=m+n, m=n \Rightarrow m=n=2$.

So Eq.(3.8)-(3.9) can be rewritten as

$u(\xi)=a_2G^2+a_1G+a_0, a_2\neq 0$ (3.10)

$v(\xi)=b_2G^2+b_1G+b_0, b_2\neq 0$ (3.11)

$a_2, a_1, a_0, b_2, b_1, b_0$ are constants to be determined later.

Substituting (3.10)-(3.11) into (3.6)-(3.7) and collecting all the terms with the same power of G together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

For Eq.(3.6)

$G^0: -ca_0-g_1-3a_0b_0=0$

$G^1: -3a_0b_1-3a_1b_0+k^2a_1\lambda^2-ca_1=0$

$G^2: -3k^2\mu a_1\lambda-ca_2-3a_0b_2-3a_2b_0+4k^2a_2\lambda^2-3a_1b_1=0$

$G^3: -10a_2\lambda k^2\mu+2a_1\mu^2k^2-3a_2b_1-3b_2a_1=0$

$G^4: 6k^2a_2\mu^2-3a_2b_2=0$

For Eq.(3.7)

$G^0: a_0-g_2-b_0=0$

$G^1: a_1-b_1=0$

$G^2: a_2-b_2=0$

Solving the algebraic equations above, yields:

$a_2=2k^2\mu^2, a_1=-2k^2\mu\lambda, a_0=a_0,$

$b_2=2k^2\mu^2, b_1=-2k^2\mu\lambda, b_0=b_0,$

$g_1=3a_0^2-a_0k^2\lambda^2, g_2=a_0-b_0$

$k=k, c=-3a_0-3b_0+k^2\lambda^2,$ (3.12)

where k, a_0, b_0 are arbitrary constants, and. $k\neq 0$.

Substituting (3.12) into (3.10)-(3.11), we obtain

$u(\xi)=2k^2\mu^2G^2-2k^2\mu\lambda G+a_0$ (3.13)

$v(\xi)=2k^2\mu^2G^2-2k^2\mu\lambda G+b_0$ (3.14)

Combining with Eq. (2.2), wecan obtain the traveling wave solutions of (3.1)-(3.2) as follows:

$$u(\xi)=2k^2\mu^2\left(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}}\right)^2-2k^2\mu\lambda\left(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}}\right)+a_0 \quad (3.15)$$

$$v(\xi)=2k^2\mu^2\left(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}}\right)^2-2k^2\mu\lambda\left(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}}\right)+b_0 \quad (3.16)$$

On the other hand, from (3.3) and (3.12) we have

$\xi=k[x+y-(-3a_0-3b_0+k^2\lambda^2)t]$ (3.17)

So combining (3.15)-(3.17) we obtain

$$u(\xi)=2k^2\mu^2\left(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda k[x+y-(-3a_0-3b_0+k^2\lambda^2)t]}}\right)^2-2k^2\mu\lambda\left(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda k[x+y-(-3a_0-3b_0+k^2\lambda^2)t]}}\right)+a_0$$

(3.18)

$$v(\xi)=2k^2\mu^2\left(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda k[x+y-(-3a_0-3b_0+k^2\lambda^2)t]}}\right)^2-2k^2\mu\lambda\left(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda k[x+y-(-3a_0-3b_0+k^2\lambda^2)t]}}\right)+b_0$$

(3.19)

Remark : Our results (3.18) and (3.19) are new exact trav-eling wave solutions for Eq.(3.1).

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IV. CONCLUSION

We have seen that some new traveling wave solutions of (2+1) dimensional breaking soliton equation are successfully found by using the Bernoulli sub-ODE method. The main pointsof the method are that assuming the solution of the ODE reduced by usingthe traveling wave variable as well as integrating can be expressed by an m -th degree polynomial in G , where $G = G(\xi)$ is the general solutions of aBernoulli sub-ODE equation. The positive integer m can be determined bythe general homogeneous balance method, and the coefficients of the polynomialcan be obtained by solving a set of simultaneous algebraic equations. The Bernoulli Sub-ODE method can be applied to many other nonlinear problems.

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