

“A Comparative study of calculating square roots using Heron’s formula and a novel method discovered

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Date of Submission: 01-01-2023

Date of Acceptance: 10-01-2023

I. INTRODUCTION

Every real number has two square roots. The principal square root of most numbers is an irrational number with an infinite decimal expansion. As a result, the decimal expansion of any such square root can only be computed to some finite-precision approximation. However, even if we are taking the square root of a perfect square integer, so that the result does have an exact finite representation, the procedure used to compute it may only return a series of increasingly accurate approximations.

The first algorithm used for approximating \sqrt{s} is known as the Babylonian method, despite there being no direct evidence beyond informed conjecture that the eponymous Babylonian mathematicians employed this method. The method is also known as Heron's method, after the first-century Greek mathematician Hero of Alexandria who gave the first explicit description of the method in his AD 60 work *Metrika*.

Author has designed a new formula which calculates square root of any integer. Error in the formula is calculated using rough estimation method and then added in the newly designed formula to minimize the error and calculate square root of an integer.

Author while studying Heron’s Formula in grade IX was motivated to design this new formula wherein Heron’s formula is used to compute error in the newly designed formula.

1.1 TITLE

“A Comparative Study of calculating square roots using Heron’s formula and a novel method discovered by Mst Ayush Pardeshi”

1.2 NEED OF THE STUDY:

1) Heron’s Formula algorithm works equally well in the p-adic numbers, but cannot be used to identify real square roots with p-adic square roots; one can, for example, construct a sequence of rational numbers by this method that converges to $+3$ in the reals, but to -3 in the 2-adics. Hence study is needed to test the values of designed formula.

2) The method employed to find square roots depends on what the result is to be used for (i.e. how accurate it has to be), how much effort one is willing to put into the procedure, and what tools are at hand. The methods may be roughly classified as those suitable for mental calculation, those usually requiring at least paper and pencil, and those which are implemented as programs to be executed on a digital electronic computer or other computing device. Algorithms may take into account convergence (how many iterations are required to achieve a specified precision), computational complexity of individual operations (i.e. division) or iterations, and error propagation (the accuracy of the final result). Hence it was necessary to design and test a new formula to get approximate values of square root of any integers.

3) Heron’s Method is iterative to generate a new method to solve the square root of an integer following study was conducted to test validity of newly designed formula.

1.3 Statement of the Problem

“A Comparative Study of calculating square roots using Heron’s formula and a novel method discovered by Mst Ayush Pardeshi”

- Can the designed formula estimate approximate square roots to minimum 2 decimal places?

- Can errors be minimized by estimating error using Heron's Formula and substituting the newly calculated value in the designed formula?

1.4 Objectives

1. To design a new formula to calculate square root of an integer.
2. To test validity of newly designed formula.
3. To analyse data and test validity of newly designed formula.
4. To graphically analyse the square roots of an integer using newly designed formula.
5. To compute error in the newly designed formula using Heron's Method.
6. To add error and calculate approximate value of square root of an integer.
7. To compare the calculated values of formula designed by Mst Ayush Pardeshi and Heron's formula.

1.5 Assumptions

The present study will be based on following assumptions,

1. Find square root of an integer nearest perfect square needs to be assumed.
2. Compute the difference between nearest perfect square and the number whose square root is to be calculated.
3. Approximate the value to first decimal place to get more accurate and precise value.
4. Accuracy is obtained by computing error using Heron's Formula.

1.6 Hypothesis

The following hypothesis are framed:

1. There is no significant difference between the calculated values of square roots using first iteration of Heron's formula and the calculated value by Mst Ayush Pardeshi's formula.
2. There is no significant difference in the validity test of calculated values of square roots using Mst Ayush Pardeshi's formula and using Heron's formula.
3. There is no significant difference in the error calculated by Mst Ayush Pardeshi's formula and in the first iteration of Heron's formula to find square root of a number.

1.6 Operational Definition:

1. Square root:

The square root of a number is the number times itself. Or we can say when we multiply a number to itself, then to regain the original number, we have to find its square root. If

p is a positive integer, then the square root of p is represented by \sqrt{p} , such that $\sqrt{p} = q$.

2. Square root by Heron's Formula:

Heron's Method is iterative. It takes an approximate guess and returns a new approximate result that is better than the first one. This can be repeated over and over until the desired accuracy is reached.

$y = 1/2(a+x/a)$ where let Y be the value of the square root and so here's Heron's formula y equals $1/2$ times a plus x over a where. x is the nonperfect square and a is the closest perfect square to x and $y = \sqrt{x}$

3. Square root Method by Mst Ayush Pardeshi:

Mst. Ayush Pardeshi's method defines square root of an integer z as

$= y = \sqrt{z} = x - \frac{n}{2x}$ where $n = |S-z|$ and S is a nearest perfect square and

$\sqrt{S} = x$

1.7 Scope and Delimitations:

Scope:

1. The study is about comparing Heron's formula with Mst. Ayush Pardeshi's formula.
2. It is applicable for square roots.

Delimitations:

1. Accuracy of the formula designed by Mst. Ayush Pardeshi is restricted to first iteration of Heron's formula.
2. Errors in the formula is to be computed by Heron's formula and need to be subtracted from the value of square root obtained by Mst. Ayush Pardeshi's formula.
3. The formula provides accuracy till 1st decimal place, Precision is to be calculated by adding error in it.
4. It is applicable only for square roots.

II. METHODOLOGY

2.1 Research Methodology:

The researcher has used Experimental Method to compare the values of square root by Heron's Formula and by Mst Ayush Pardeshi's method. This collected data is in the form of arithmetic numbers so the researcher used the following statistical technique to interpret the data,

- The measure of Central Tendency
- T-Test
- Measure of Variability

2.2 Research Design:

In this study, the researcher will use an experimental method for comparing the values of square root by Heron’s Formula and by Mst Ayush Pardeshi’s method.

For this purpose, simple random sampling is used.

2.2.1 Variables:

Dependent Variable	Independent Variable	Extraneous Variable
S is a nearest perfect square and $\sqrt{S} = x$	square root of an integer z as $y = \sqrt{z}$	Errors computed by Heron’s formula

2.2.2 Population:

The researcher will consider square roots of an integer as his population.

2.2.3 Sample:

The researcher will use the Simple Random Sample Technique for the selection of samples from the population. He will consider integers from 1 to 100 as his sample.

2.2.4 Statistical Techniques:

This collected data is in the form of arithmetic numbers so the researcher used the following statistical technique to interpret the data,

- The measure of Central Tendency
- T-Test

III. DISCUSSION

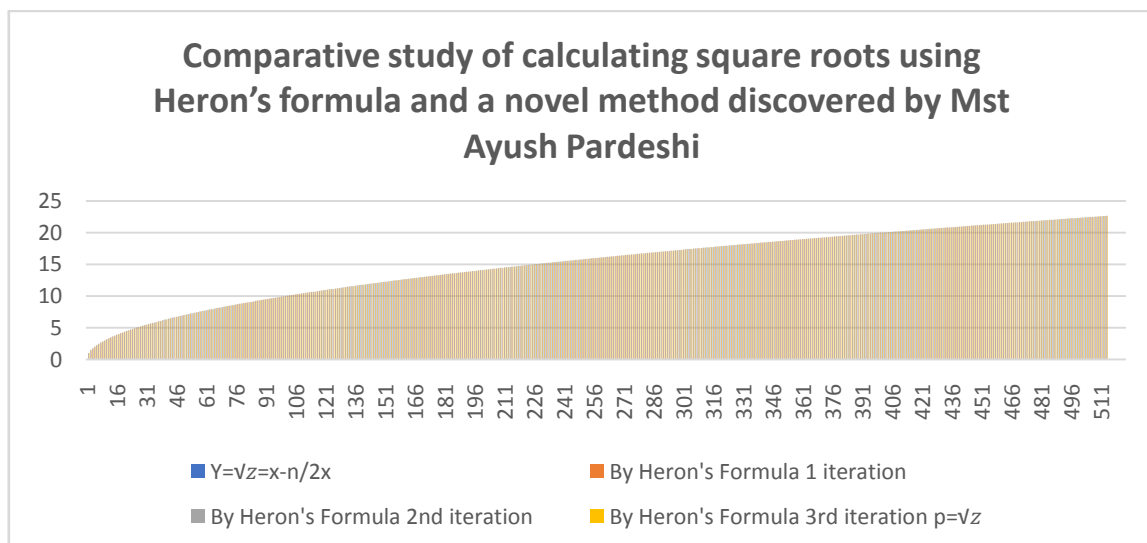
3.1 DATA COLLECTION, ANALYSIS, AND INTERPRETATION:

3.1.1

Data Collection : Link of Spreadsheet containing sample.

<https://docs.google.com/spreadsheets/d/14og-sKg64EkOVkVA-5QIIMoX-MTWvKM9RbsuCdSnEWE/edit?usp=sharing>

3.1.1.i) Graphical Representation of Comparative values of square roots by Ayush Pardeshi’s Method and by Herons formula for 1st, 2nd, and 3rd iteration.



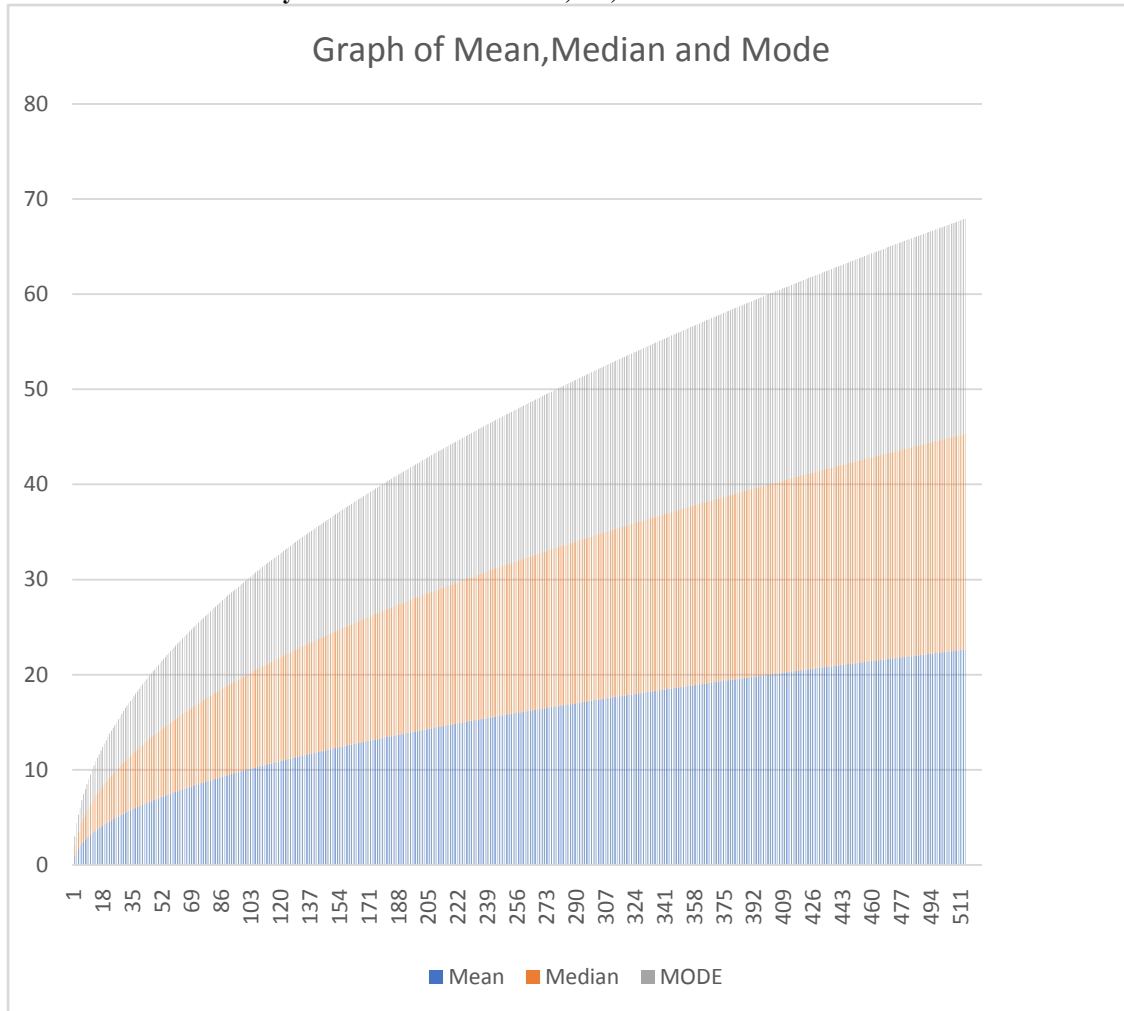
3.1.2

Interpretation:

- There is no significant difference in the values of square roots by Mst.AyushPardeshi's Method and by Heron's formula for first iteration.
- There is an error computed in square root by Ayush Pardeshi's method and by Heron's second and third iteration.
- Value of Ttest is $0.5 < p$ value.
- Hence there is no significant difference between the calculated values of square roots using first iteration of Heron's formula and the calculated value by Mst Ayush Pardeshi's formula.
- There is no significant difference in the error calculated by Mst Ayush Pardeshi's formula and in the first iteration of Heron's formula to find square root of a number.

3.1.1.ii.

Graphical Representation of Comparative values of Mean, Median and Mode of square roots by Ayush Pardeshi's Method and by Herons formula for 1st, 2nd, and 3rd iteration.



3.1.3

Interpretation:

- Mean, Median and Mode fall on same line.
- Hence there is no significant difference between the calculated values of square roots using first iteration of Heron's formula and the calculated value by Mst Ayush Pardeshi's formula.
- There is no significant difference in the error calculated by Mst Ayush Pardeshi's formula and in the first iteration of Heron's formula to find square root of a number.

3.2 Analysis

- Mean = Sum of all observations/Number of observations= $M = \frac{\sum_{i=1}^n x_i}{n}$

Where x is data of i th observation, n is number of observation.

- Median=The value of the **middlemost observation**, obtained after arranging the data in ascending or descending order, is called the **median** of the data.
- Mode using the following formula:

$$\text{Mode} = l + \left[\frac{f_m - f_1}{2f_m - f_1 - f_2} \right] * h$$

l = lower limit of modal class,

f_m = frequency of modal class,

f_1 = frequency of class preceding modal class,

f_2 = frequency of class succeeding modal class,

h = class width

- T-Test

The formula for the two-sample t-test

$$t = \frac{x_1 - x_2}{\sqrt{S^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

In this formula, t is the t -value, x_1 and x_2 are the means of the two groups being compared, s_2 is the pooled standard error of the two groups, and n_1 and n_2 are the number of observations in each of the groups.

A larger t -value shows that the difference between group means is greater than the pooled standard error, indicating a more significant difference between the groups.

IV. CONCLUSION

1. There is no significant difference between the calculated values of square roots using first iteration of Heron's formula and the calculated value by Mst Ayush Pardeshi's formula.

2. There is no significant difference in the validity test of calculated values of square roots using Mst Ayush Pardeshi's formula and using Heron's formula.

3. There is no significant difference in the error calculated by Mst Ayush Pardeshi's formula and in the first iteration of Heron's formula to find square root of a number.

4. Error is computed with following formula

$= y = \sqrt{z} = x - \frac{n}{2x}$ where $n = |S - z|$ and S is a nearest perfect square and

$$\sqrt{S} = x$$

Error $\mathcal{E} =$

$$x_3 = \frac{1}{2} \left(x_2 + \frac{z}{x_2} \right)$$

ACKNOWLEDGEMENT

- I would like to thank our parents for always being our motivators.
- I would like to thank our respected Principal sir, Mr. Nischal Nair for providing us this opportunity to participate in this event.
- I would like to thank our teacher (Ms. Sanchita Jadhav) for her valuable guidance in this project.

We are grateful to IRIS NATIONAL FAIR in collaboration with CBSE.

REFERENCES

- [1]. Brown, P.R. (2019). Detecting square numbers. *Quaestiones Mathematicae*, [online]. Available at: <https://doi.org/10.2989/16073606.2019.1678530>
- [2]. Bernstein, D.J. (1998). Detecting perfect powers in essentially linear time. *Mathematics of Computation*, 67(223), 1253-1283.
- [3]. Link of Spreadsheet containing sample.
- [4]. <https://docs.google.com/spreadsheets/d/14og-sKg64EkOVkVA-5QIIMoX-MTWvKM9RbsuCdSnEWE/edit?usp=sharing>