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Newton's Law of Cooling by "Emad–Falih Transform."

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ABSTRACT
In this paper, we use Emad-Falih transform to find the solutions of Newton's law cooling. Further, some problems based on Newton's law of cooling are solved.

Keywords: Newton's law of cooling, Emad-falih integral transform.

I. INTRODUCTION :

Recently, integral transforms are one of the mostly used simple mathematical technique to obtain the solutions of advance problems of space, science, technology, engineering, commerce and economics. The important feature of these integral transform is to provide exact solution of the problem without lengthy calculations.

Due to this important feature of the integral transforms many researchers are attached to this field and are engaged in introducing various integral transforms. Recently , in September 2021, Kushare and Patil [1] introduced Kushare transform for facilitating the process of solving differential equations in time domain. Further , in October 2021, Khakale and Patil [2] introduced Soham transform. As researchers are introducing the new integral transforms, at the same time they are also interested in applying the transforms to various fields, various equations in different domains. In January 2022, Sanap and Patil [3] used Kushare transform to solve the problems on Newton's law of cooling. In April 2022, D. P. Patil [4] used Kushare transform for solving the problems on growth and decay. In October 2021, D. P. Patil [5] used Sawi transform in Bessel functions. Further, Patil [6] used Sawi transform 0 error functions to evaluate improper integrals. Laplace transform and Snehu transforms are used by Patil [7] in chemical sciences. Patil [8] solved wave eqations by using Sawi transform and its convolution theorem using Mahgoub transform. Parabolic boundary value problems are solved by D. P. Patil [9] . Solutions of wave equation is obtained by using double Laplace and double Sumudu transforms by D. P. Patil [10]. Dr. Patil [11] also obtained dualities between double integral transforms. Laplace, Elzaki and Mahgoub transforms are compared and used for solving system of first order and first degree by Kushare and Patil [12]. D. P. Patil [13] usedAboodh and Mahgoub transform for solving boundary value problems of the system of ordinary differential equations. Double Mahgoub transformed is used by Patil [14] to solve parabolic boundary value problems. In 2018, D. P. Patil [15] comparatively study Laplace, Sumudu, Aboodh, Elazki and Mahgoub transform and used it for solving boundary value problems. Recently, in March 2022, Dipali Kaklij [16] introduced double new general integral transform. Patil [17] used Soham transform for solving Volterra integral equations of first kind. Further, Patil with Tile and Shinde [18] used transform for solving Volterra integral equations of first kind. Dinkar P. Patil et al [20] used Soham transform to solve the system of differential equations of first order. D. P.Patil et al [21] used general integral transform of error functions to evaluate the improper integrals. Recently D. P.Patil et al [22] used double general integral transform for the solution of parabolic boundary value problems

In this paper we use Emad- Falih transform introduced by Emad Kuffi and Sara Falih Maktoof [19] to obtain the solution of Newton's law of cooling.[1]

II. DEFINATION OF EMAD-FALIH INTEGRAL TRANSFORM :

The proposed integral transform is defined for an exponential order function :

B = {f(t): ⍰K, m1, m2 > 0,|f(t)|< $Ke^{m^{2}j|t|}$ if t ϵ $(-1)^{j}$ X [0, ∞)} (1)

Where : f(t) is a function in the B set, K is a finite constant number, m1 and m2 may or may not be finite.

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The kernel function of Emad – Falih integral transform symbolized by (EF) is defined by the integral equation :

$$EF\{f(t)\} = T(\phi) = \frac{1}{\varphi}\int_0^{\infty} e^{-\varphi^2 t} f(t)\, dt \qquad (2)$$

Where $t \geq 0$, $m1 \leq \phi \leq m2$ and ϕ is a variable that is used as a factor to the variable t in the function f

2.1 Emad –Falih Transform of some fundamental functions:

Now we will state some Emad Falih integral transform of some elementary functions

Sr. No.	**Functions**	**Emad-Falih Transform**
1.	K	$\frac{k}{\varphi^2}$
2.	t^n	$\frac{n!}{\varphi^{2n+3}}$
3.	e^{at}	$\frac{1}{\varphi(\varphi^2 - a)}$
4.	sin at	$\frac{1}{\varphi(\varphi^4 + a^2)}$
5.	cos at	$\frac{\varphi}{(\varphi^4 + a^2)}$
6.	sinh at	$\frac{a}{\varphi(\varphi^4 - a^2)}$
7.	cosh at	$\frac{\varphi}{(\varphi^4 - a^2)}$

2.2 Inverse Emad-Falih Transform :

This table includes inverse Emad Falih integral transform of some functions

Sr.No.	Function	Inverse Emad-Falih transform
1.	$\frac{k}{\varphi^2}$	K
2.	$\frac{n!}{\varphi^{2n+3}}$	t^n
3.	$\frac{1}{\varphi(\varphi^2 - a)}$	e^{at}
4.	$\frac{1}{\varphi(\varphi^4 + a^2)}$	sin at
5.	$\frac{\varphi}{(\varphi^4 + a^2)}$	cos at
6.	$\frac{a}{\varphi(\varphi^4 - a^2)}$	sinh at
7.	$\frac{\varphi}{(\varphi^4 - a^2)}$	cosh at

2.3 Emad-Falih Transform of derivative of the function f(t) :

Let T(ϕ) is the Emad-Falih integral transform of [EF (f(t) = T(ϕ))]

Then, The New integral Transform "Emad-Falih" is,

$EF[f'(t)] = \frac{-f(0)}{\varphi} + \phi^2\, T(\phi)$

2.4 Applying EF transform into first-order ordinary differential equations :

Consider the linear first-order ordinary differential equation

$\frac{dy}{dt} + py = f(t), > 0$ with the initial condition $y(0) = a$,

where a and p are constants and f(t) is an external input function so that EF(f(t)) transform exists.

By applying Emad-Sara integral transform on the above equation , We get ,

$T(\phi) = \frac{\bar{f}(\alpha)}{(p+\varphi^2)} + \frac{a}{\varphi(p+\varphi^2)}$

III. APPLICATION OF EMAD-FALIH TRANSFORM IN NEWTON'S LAW OF COOLING.

The newton's law of cooling is stated by following equation,

$\frac{dT}{dt} = -C\,(T - T_e)$ ….[1]

with initial condition at, $T(t_0) = T_0$

Where, T is tempreture of the object ,

T_e is the constant temperature of the environment (surroundings),

T_0 is the initial temperature of the object at time t_0 and C is the constant of proportionality.

Let, taking Emad-falih integral transform on both sides of [1], We get,

$EF\left\{\frac{dT}{dt}\right\} = EF\,[-C\,(T - T_e)\,]$

$\therefore \frac{-T(0)}{\varphi} + \varphi^2\,.T(\phi) = -\,C.EF\,(T) + C\,.\,EF\,(T_e)$

$\Rightarrow \frac{-T(0)}{\varphi} + \varphi^2\,.T(\phi) = -\,T(\phi) + C\,.T_e\ EF(1)$

Here , $t = 0$,$T = T_0$,

$\frac{-T_0}{\varphi} + \varphi^2\,.T(\phi) = -C\,.T(\phi) + C.\,T_e\frac{1}{\varphi^3}$

$\Rightarrow \phi^2\,.T(\phi) + C.\,T(\phi) = \frac{T_0}{\varphi} + C.\,\frac{T_e}{\varphi^3}$

$\therefore T(\phi) = T_e\left[\frac{C}{\varphi^3(\varphi^2+C)}\right] + T_0\left[\frac{1}{\varphi(\varphi^2+C)}\right]$

By using partial fractions method we get,

$\Rightarrow \frac{C}{\varphi^3(\varphi^2+C)} = \frac{1}{\varphi^3} - \left[\frac{1}{\varphi(\varphi^2+C)}\right]$

$\therefore T(\phi) = T_e\left\{\frac{1}{\varphi^3} - \left[\frac{1}{\varphi(\varphi^2+C)}\right]\right\} + T_0\frac{1}{\varphi(\varphi^2+C)}$

$\therefore T(\phi) = \frac{T_e}{\varphi^3} - \frac{T_e}{\varphi(\varphi^2+C)} + \frac{T_0}{\varphi(\varphi^2+C)}$

$\therefore T(\phi) = T_e\,.\frac{1}{\varphi^3} + (T_0 - T_e)\left[\frac{1}{\varphi(\varphi^2+C)}\right]$

Now, By applying inverse EF Transform on both sides, we get,

$EF^{-1}[\,T(\phi)\,] = EF^{-1}\left[\frac{T_e}{\varphi^3}\right] + EF^{-1}\left[(T0 - Te)\left[\frac{1}{\varphi(\varphi^2+C)}\right]\right]$

$\therefore T(t) = T_e\,.\,EF^{-1}.\left[\frac{1}{\varphi^3}\right] + (T_0 - T_e)EF^{-1}\left[\frac{1}{\varphi(\varphi^2+C)}\right]$

$\therefore T(t) = T_e + (T_0 - T_e)\,e^{-ct}$

In this section, we solve some problems based on Newton's law of cooling by using EF integral Transform,

IV. APPLICATIONS:

Problem 1 : A hot coffee with initial tempreture of 115^0 F kept in a room tempreture of 35^0 F . The rate of change of tempreture is 20^0 F per/min. how long it will take coffee to cool to a tempreture of 40^0 F?

⇒ Assuming that coffee obeys Newtion's law of cooling,

$\frac{dT}{dt} = -C\,(T - 35)$, $T(0) = 115$, $T'(0) = -20$

First we will find C by using initial conditions,

$-20 = -C\,(115 - 35)$

$\therefore C = 0.25$

So, the differential equation can be written as,

$\frac{dT}{dt} = -0.25\,(T - 35)$

By, applying EF Transformation, we get,

$EF\left\{\frac{dT}{dt}\right\} = -0.25\,EF\,(T - 35)$

$\Rightarrow \frac{-T(0)}{\varphi} + \varphi^2\ T\,(\phi) = -0.25\ T(\phi) + 0.25 \times 0.35\ EF(1)$

$\therefore T(\varphi)\,[\varphi^2 + 0.25] = 0.25 \times 0.35\left\{\frac{1}{\varphi^3}\right\} + \frac{115}{\varphi}$

$\therefore T(\phi) = 35\left\{\frac{0.25}{\varphi^3(\varphi^2+0.25)}\right\} + 115\left\{\frac{1}{\varphi(\varphi^2+0.25)}\right\}$

By, using partial fraction, we get ,

$T(\phi) = \frac{35}{\varphi}\left\{\frac{1}{\varphi^2} - \frac{1}{(\varphi^2+0.25)}\right\} + 115\left\{\frac{1}{\varphi(\varphi^2+0.25)}\right\}$

$\therefore T(\phi) = 35\left(\frac{1}{\varphi^3}\right) + 80\left\{\frac{1}{\varphi(\varphi^2+0.25)}\right\}$

By applying EF inverse transformation ,

$T(t) = 35 + 80\,.\,e^{-0.25t}$

Putting T = 40 in above equation,

$40 = 35 + 80\,.\,e^{-0.25t}$

$e^{-0.25t} = \frac{1}{16}$

$e^{0.25t} = 16$

Taking log on both sides, we get,

$0.25\,t = l\,16$ $\therefore t = 11.09354888$ min.

∴ Coffee will take 11.09 minutes for cooling to a temperature of 40^0 F.

Problem 2 : A heated metal beam cools at the rate of 3^0 C per/min when its tempreture is 50^0C .find the time taken to cool at 36^0C if the temperature of surroundings is 27^0C .

⇒ Assuming that a heated metal beam obeys Newton's law of cooling, we have,

$\frac{dT}{dt}$= - C (T - 27) , T(0) = 50 , T′(0) = - 3,

First, we will find value ofC by using initial condition,

- 3 = - C (50 – 27)

∴ C = 0.13

The differential equation can be written as,

$\frac{dT}{dt} = -0.13\,(T-27)$

Now, by EF Transform , we get ,

$EF\left\{\frac{dT}{dt}\right\} = EF\{-0.13\,(T-27)\}$

$\therefore \frac{-T(0)}{\varphi} + \varphi^2\, T(\varphi) = -0.13\, EF(T) + 0.13 \times 27\, EF(1)$

$\therefore \frac{-50}{\varphi} + \varphi^2\, T(\varphi) + 0.13\, T(\varphi) = 0.13 \times 27\, EF(1)$

$\therefore T(\phi)\,[\varphi^2 + 0.13] = 0.13 \times 27 \left(\frac{1}{\varphi^3}\right) + \left(\frac{50}{\varphi}\right)$

$\therefore T(\phi) = 27\left\{\frac{0.13}{\varphi^3(\varphi^2+0.13)}\right\} + 50\left\{\frac{1}{\varphi(\varphi^2+0.13)}\right\}$

$\therefore T(\phi) = 27\left\{\frac{1}{\varphi^2} - \frac{1}{(\varphi^2+0.13)}\right\} + 50\left\{\frac{1}{\varphi(\varphi^2+0.13)}\right\}$

$\therefore T(\phi) = 27\left\{\frac{1}{\varphi^3}\right\} + 23\left\{\frac{1}{\varphi(\varphi^2+0.13)}\right\}$

BY taking inverse EF transform,

$T(t) = 27 + 23\, e^{-0.13t}$

Putting T = 36 in above equation, we get ,

$36 = 27 + 23\, e^{-0.13t} \Rightarrow e^{-0.13t} = \frac{9}{23}$

$e^{0.13t} = \frac{23}{9}$

Taking log on both sides, we get,

$0.13\,t = \ln 2.56 \Rightarrow 0.13\,t = 0.9400$

t = 7.23 min.

∴ A heated metal beam will take 7.23 minutes for cooling to a temperature of 36^0 C .

V. CONCLUSION:

We Sucessfully used "Emad – Falih Transform" to solve the problems based on Newton's Law of cooling .

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