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Control of spatial solitons in photovoltaic photo refractive materials

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ABSTRACT:Spatial solitons are formed in photovoltaic-photorefractive materials where diffraction of an optical beam is exactly compensated by nonlinear self-focusing due to the photovoltaic field and electro-optic effect. These solitons may have steady-state irradiances of microwatts to milliwatts per square centimeter and widths as small as 10μm in lithium niobate. Soliton We need to investigate MNLSE using linear stability analysis which gives us the growth rate instability and the parameter domain where optical control of spatial solitons takes place. In this paper, we have to describe how refractive index changes when a light beam is propagating in photovoltaic-photorefractive materials or optical fiber. The propagation of light beam in nonlinear media leads to the formation of solitons (temporal or spatial solitons)[1]. From space charge field and nonlinear Helmholtz wave equation, we re-derive Modified nonlinear Schrödinger equation (MNLSE). The MNLSE contains the nonlinearity parameters α, ρ and β which may cause the formation of spatial solitons and the parameter δ influencing the bending of the trajectory of solitons. We need to use linear stability analysis to study the instability due to the parameters α and ρ. We have to set some criterial under which a given perturbation leads to the modulation instability in the system.

KEYWORDS: Spatial solitons, perturbation, non-linear media, instability, Modified non-linear Schrödinger equation.

I. INTRODUCTION

In the middle of the 19th century, Zabusky and Kruskal(Miki, 2001) gave the name solitary waves to particles which can undergo elastic collision and their speeds, wavelengths, amplitudes and shapes remain invariant[2]. This new theory attracted many scientists to carry out the research about it. Solitons were found to be solutions of integrable partial differential equations applicable in communications, electronic devices, etc.In the context of optics, this type of light waves propagating a long distance without losing their shape and initial behaviours are also used in optical fibers to carry information over a long distance with energy loss low compared to copper cables. There are two different kinds of solitons depend on the balance between nonlinearity effect and diffraction or dispersion phenomenon in that media. When the nonlinearity effect balances the dispersion phenomenon, soliton formed are temporal in that case is known as temporal soliton but when nonlinearity effect balances the diffraction phenomenon a soliton formed is spatial solitons[3]. The spatial soliton wave moves in nonlinear media keeping its dimension and its shape invariant (see Fig. 1). In contrast to normal light (Fig. 2), it keeps all its original physical behaviourswhen it is propagating in inhomogeneous nonlinear media[4]. When optical soliton is propagating in fiber, it is a solitary wave governed by the nonlinear Schrödinger equation (NLSE) and its variant[5]. This curious special optical beams that propagate without diffraction attracted many researchers to improve their knowledge about it. A soliton wave is different from the natural electromagnetic wave[1].

Fig. 1 propagation of optical soliton.

(Fig. 1)Propagation of optical solitons moving in homogeneous linear medium, where it can diffract in different directions, reflect and refract according to the material it hits and depending on the material refractive index. Among the nonlinear

Fig. 2 Light diffracts when it is propagating in linear homogeneous media [4]

media which cause the formation of solitons are photorefractive materials. In 1965,

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Arthur Ashkin and his colleagues were dealing with experiments in their laboratories, using lithium niobate and laser beam to convert laser light (which is intense) from one color to another color using the second harmonic generation process[6],[7]. Photovoltaic-photorefractive materials present the change in refractive index when light beam is applied on them[8]. This makes the materials a support of solitons. In this section, we want to re-derive the modified nonlinear Schrödinger equation which is the model equation governing the solitons in Photovoltaic-photorefractive materials. For this, starting with the continuity equation for charges and Maxwell's equations, we obtain the wave and Helmholtz equations. Considering a laser field with space dependent amplitude, we exploit the space-dependence of the refractive index change to obtain the modified nonlinear Schrödinger equation. The linear stability analysis is used to examine the modulation instability in the system, and then to get the instability gain as well as the parameter domain where solitons may be formed. Finally, we present the numerical tool that may help to perform direct simulation of the model equation.

II. THEORETICAL MODEL

2.0 Introduction

Photovoltaic-photorefractive materials present the change in refractive index when light beam is applied on them. This makes the materials a support of solitons. In this section, we want to re-derive the modified nonlinear Schrödinger equation which is the model equation governing the solitons in photovoltaic-photorefractive materials. For this, starting with the continuity equation for charges and Maxwell's equations, we obtain the wave and Helmholtz equations.

2.1 Space charge field

Space charge field. The space charge field is produced after applying the photon on photovoltaic photorefractive material where the charge space distribution takes place with dependence on intensity of light applied[9].

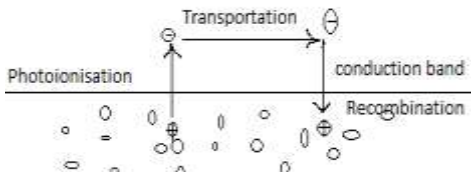

Fig. 3 Space charge field formation [10]

When a light beam illuminates the photovoltaic-photorefractive material (photoionization), a photon causes the electrons to move from valence band to the conduction band where there is own of electrons. When illumination is removed on the material, recombination takes place. The field between photoionization and recombination (see Fig. 3) is space charge field. The induced space charge field Esc can be obtained from the standard set of rate, donor ionization rate equation, electro continuity equation, current density (J) equation.

Continuity equation for immobile donors = generation (GR) minus recombination (R)[11].

$$\frac{\partial}{\partial t}N_D^+ = G_R = (\mathrm{s}_i \mathrm{I} + B_r)(N_D - N_D^+) - \gamma_R N N_D^+ \quad (1)$$

Where N^+_D, si, I, B_T, N_D and γ_R, Nare densitiesof ionized donor, photoexcitation cross section, optical intensity, dark generation rate, total donor density, recombination rate and density of electrons[7].

Continuity equation of electrons

$$\frac{\partial}{\partial t}N = \frac{\partial}{\partial t}N_D^+ + \frac{1}{e}\vec{\nabla}.\vec{J} \quad (2)$$

Gauss'lawgives

$$\vec{\nabla}.\varepsilon\vec{E} = \rho = e(N_D^+ - N_A - N) \quad (3)$$

Current density equation = Drift + Diffusion + photogalvanic[11]

$$\mathrm{J} = \mathrm{e}\mu_e EN + k_B T \mu_e \nabla N + J_{ph} \quad (4)$$

Where $J_{ph} = k_{si}(N_D - N_D^+)I$, and I is the intensity of light, e, E_{sc}, k_B, T, ε, J and N_A are densities of electron, electron charge, space charge field, Boltzmann constant, temperature, dielectric constant, current density and acceptor density, respectively. At constant illumination, we have steady state equations, where time constant, implying that $\frac{\partial}{\partial t}N = 0$, $\frac{\partial}{\partial t}N_D^+ = 0$ and I_d is the irradiance intensity[7].

From Eq. (1), $N = \dfrac{si(I+I_d)\left(N_D - N_D^+\right)}{\gamma_R N_D^+}$ (5)

From Eq. (2), $N_D^+ = \dfrac{\varepsilon_0\varepsilon_r}{e}\dfrac{\partial E_{sc}}{\partial x} + N_A + N$ (6)

Introducing Eq. (6) into Eq. (5), we get,

$$N = \frac{si\left(I + I_d\right)\left(N_D - N_A\left(\frac{\varepsilon_0\varepsilon_r}{eN_A}\frac{\partial E_{sc}}{\partial x} + 1\right) + N\right)}{\gamma_R N_A\left(\left(\frac{\varepsilon_0\varepsilon_r}{e}\frac{\partial E_{sc}}{\partial x} + 1\right) + N\right)} \quad (7)$$

International Journal of Advances in Engineering and Management (IJAEM)
Volume 3, Issue 5 May 2021, pp: 559-564 www.ijaem.net ISSN: 2395-5252

If $N \square N_A, N \square N_A, N \square N_D^+$ and $N_A \square N_D^+$, this implies that N in right hand side of Eq. (7) can be neglected,

$$N = \frac{si(I+I_d)\left(N_D - N_A\left(\frac{\varepsilon_0\varepsilon_r}{eN_A}\frac{\partial E_{sc}}{\partial x}+1\right)\right)}{\gamma_R N_A\left(\frac{\varepsilon_0\varepsilon_r}{e}\frac{\partial E_{sc}}{\partial x}+1\right)} \quad (8)$$

The beam intensity varies slowly with respect to the coordinate x that is why $\frac{\partial E_{sc}}{\partial x} \approx o$. Then the terms containing $\frac{\partial E_{sc}}{\partial x}$ are very small as follows $\frac{\varepsilon_0\varepsilon_r}{eN_A}\frac{\partial E_{sc}}{\partial x} \square 1$.

The Eq. (8) becomes

$$N = \frac{si(I+I_d)(N_D - N_A)}{\gamma_R N_A} \quad (9)$$

Where there is constant illumination. Thus intensity $I_{(x,z)}$ becomes constant value, $x \to \pm\infty$ that is $I_{(x\to\pm\infty)} = I_\infty$ and $E_{sc}(x \to \pm\infty) = E_0$. This leads to

$$N_0 = \frac{si(I_\infty + I_d)(N_D - N_A)}{\gamma_R N_A} \quad (10)$$

At the crystal borders (edges) $x \to \pm\infty$, the intensity of light is constant and does not depends on position x. then, $J_{0=N_0 e\mu_e E_0}$, where $N(x \to \pm\infty) = N_0$ and $E(x \to \pm\infty) = E_0$, the quantity E_0 which is external field to the photorefractive crystal and N_0 are the constants. Also it is known that $J_{(x)} = N_0 e\mu_e E_0$ implying that $J_{(0)} = N_0 e\mu_e E_0$, where $J_{(x)}$ is current density which depends on x while J_0 is constant current density[12]. From Eq. (4), we obtain .

$$J = e\mu_e N E_{sc} + k_B T\mu_e \frac{\partial N}{\partial x} + k_p s_i (N_D - N_D^+) I \cdot \quad (11)$$

We substitute $\frac{J_0}{e\mu_e}$ which is equal to $N_0 E_o$ as we have in Eq. (12) below:

$$N_0 E_o = \frac{J_0}{e\mu_e} = NE_{sc} + k_B T\frac{\partial N}{\partial x} + \frac{k_p s_i}{e\mu_e}(N_D - N_D^+) I \quad (12)$$

From Eq. (12), we have

$$E_{sc} = \frac{1}{N}\left(N_0 E_o - k_B T\frac{\partial N}{\partial x} - \frac{k_p s_i}{e\mu_e}(N_D - N_D^+) I\right) \quad (13)$$

Let us calculate the derivative of N with respect to x,

$$\frac{\partial N}{\partial x} = \frac{s_i(N_D - N_D^+)}{\gamma_R N_A}\frac{\partial I}{\partial x} \quad (14)$$

Replacing Eq. (14), Eq. (10) and Eq. (1) in Eq. (13) gives [12]

$$E_{sc} = \left(\frac{I_\infty + I_d}{I + I_d}\right)E_0 - \frac{k_B T}{e}\frac{1}{I+I_d}\frac{\partial I}{\partial x} - E_p\frac{I}{I+I_d} \quad (15)$$

Where $E_p = \frac{k_B \gamma N_A}{e\mu_e}$.

Let us assume that $I \square I_\infty$, i.e the intensity at the beam axis I is very high than the intensity at the edge I_∞ of the crystal, which is negligible compared to the one at the axis of the crystal. We can substitute the relation $I \approx I - I_\infty$. By replacing I its value in Eq. (15) gives[9]and[12]

$$E_{sc} = \left(\frac{I_\infty + I_d}{I + I_d}\right)E_0 - \frac{k_B T}{e}\frac{1}{I+I_d}\frac{\partial I}{\partial x} + E_p\frac{I_\infty - I}{I+I_d} \quad \textbf{(16)}$$

2.2 Wave and Helmholtz equation

Considering the linear medium, Maxwell's equations are

Gauss 'law: $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon}$ (17)

Gauss 'law: $\vec{\nabla}.\vec{B} = 0$ (18)

Faraday's law: $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ (19)

Ampere-Maxwell's law:

$$\vec{\nabla}\times\vec{B} = \mu\vec{J} + \mu\varepsilon\frac{\partial\vec{E}}{\partial t} \quad (20)$$

With $\vec{J} = e\mu_e N\vec{E} = \sigma\vec{E}$, , $\varepsilon = \varepsilon_0\varepsilon_r$ and $\mu = \mu_0\mu_r$, ε_0, ε_r, μ_0, μ_r, σ are the permittivity of free space, the relative permittivity of free space, the permeability of free space, the relative permeability of free space and the conductivity of the medium, respectively. The Ampere- Maxwell's Eq. (20) can be rewritten as

$$\vec{\nabla}\times\vec{B} = \mu\vec{J} + \mu\varepsilon\frac{\partial\vec{E}}{\partial t} \cdot \quad (21)$$

By calculating the curl of both sides of Eq. (21), we get

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$$\vec{\nabla}(\frac{\rho}{\varepsilon})-(\vec{\nabla}.\vec{\nabla})\vec{E}=-\frac{\partial}{\partial t}\left(\sigma\mu\vec{E}+\mu\varepsilon\frac{\partial\vec{E}}{\partial t}\right). \quad (22)$$

For homogenous media, we have $\vec{\nabla}(\frac{\rho}{\varepsilon})=0$ and Eq. (22) becomes

$$\nabla^2\vec{E}=\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}+\mu\sigma\frac{\partial\vec{E}}{\partial t}. \quad (23)$$

When σ,μ and ε are independent, we get

$$\nabla^2\vec{E}-\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}=\mu\sigma\frac{\partial\vec{E}}{\partial t}. \quad (24)$$

This is the wave equation in the conducting material[13]. The monochromatic waves have the same frequency and the fields have harmonic time dependence $e^{i\omega t}$. Let us take time dependence is such that $\vec{E}=E(\mathrm{x},\mathrm{y},\mathrm{z})e^{i\omega t}$. Then replacing this equation in Eq. (24) leads to

$$\nabla^2\vec{E}+k^2\vec{E}=0, \quad (25)$$

When $k^2=\omega^2\mu\varepsilon(\frac{\sigma}{i\omega\varepsilon}+1)$.

The equality k^2 is the square complex number[14]. When $\frac{\sigma}{i\omega\varepsilon}\ll 1$, then k^2 is purely imaginary and when $\frac{\sigma}{i\omega\varepsilon}\gg 1$, k^2 is real. The Eq. (25) is the Helmholtz equation. [15]When a light wave propagates in a material, its speed is $\mathrm{v}=\frac{n}{c}$, which implies that $k=\frac{\omega}{v}=\frac{n\omega}{v}=nk_0$

Let us replace k by its value in Eq. (25), we obtain

$$\vec{\nabla}\vec{E}+(\mathrm{k}_0\hat{\mathrm{n}})\vec{\mathrm{E}}=0, \quad (26)$$

where $k=nk_0$ which is wave number of the wave in material [9]. Let us assume that the field is $\phi(\mathrm{x},\mathrm{z})\,\mathrm{e}^{i(\mathrm{k}z-\omega t)}$, with $k=\frac{2\pi}{\lambda}$. The light beam is only allowed to diffract along x direction which is implying that there is no motion of light beam along y direction. The perturbed extra-ordinary refractive index (overall refractive index of the photorefractive material when the intensity of light is applied on it given as

$$\hat{n}=n_e^2-n_e^4r_eE_{sc}, \quad (27)$$

where n_e is the unperturbed extra-ordinary refractive index[14]. From nonlinear Helmholtz equation, we can derive the parabolic equation. Let us replace $\vec{E}=\hat{x}\phi(\mathrm{x},\mathrm{z})\,\mathrm{e}^{i(\mathrm{k}z-\omega t)}$ into Eq. (26), where ϕ is slowly varying amplitude of electric field. From nonlinear Helmholtz equation, we have

$$\vec{\nabla}^2\phi(\mathrm{x},\mathrm{z})\,\mathrm{e}^{i(\mathrm{k}z-\omega t)}+(k_0\,\hat{\mathrm{n}})^2\phi(\mathrm{x},\mathrm{z})\,\mathrm{e}^{i(\mathrm{k}z-\omega t)}=0 \quad (28)$$

After second derivative and replacing Eq. (27) in Eq. (28) and then divide by $\mathrm{e}^{i(\mathrm{k}z-\omega t)}$, we neglect the second derivative in z (slowly amplitude approximation), $\frac{d^2\phi}{dz^2}\ll\frac{d\phi}{dz}$, we obtain

$$\frac{\partial^2}{\partial x^2}\phi(x,z)+2ik\frac{\partial}{\partial z}\phi(x,z)-\mathrm{k}_0^2\,n_e^4r_eE_{sc}\phi(x,z)=0 \quad (29)$$

From Eq. (29), Helmholtz equation is

$$i\frac{\partial}{\partial z}\phi(x,z)+\frac{1}{2k_0n_e}\frac{\partial^2}{\partial x^2}\phi(x,z)-\frac{1}{2}k_0n_e^2r_eE_{sc}\phi(x,z)=0, \quad (30)$$

Which is a parabolic equation from nonlinear Helmholtz equation.

2.3 Modified nonlinear Schrödinger equation

The modified nonlinear Schrödinger equation can be obtained by replacing space charge field Eq. (16) into parabolic equation from Helmholtz equation Eq. (30) gives

$$i\frac{\partial}{\partial z}\phi+\frac{1}{2k_0n_e}\frac{\partial^2}{\partial x^2}\phi-\frac{1}{2}k_0n_e^2r_e\left(\left(\frac{I_\infty+I_d}{I+I_d}\right)E_0-\frac{k_BT}{e}\frac{1}{I+I_d}\frac{\partial I}{\partial x}+E_p\frac{I_\infty-I}{I+I_d}\right)\phi=0. \quad (31)$$

It leads to

$$i\frac{\partial}{\partial\xi}A+\frac{1}{2}\frac{\partial^2}{\partial s^2}A-\beta\frac{(1+\rho)}{1+|1|}A-\alpha\frac{(\rho-|A|^2)}{1+|\mathrm{A}|^2}A+\frac{\delta}{1+|A|^2}\frac{\partial|A|^2}{\partial s}=0, \quad (32)$$

where E_0, E_{ph} is an external field and photovoltaic field respectively, $\xi=\frac{z}{k_0n_ex_0^2}$, $s=\frac{x}{x_0}$, $\alpha=(k_0x_0^2)(\frac{n_0^4r_e}{2})\mathrm{E}_p$, $\beta=(k_0x_0)^2\frac{n^4r_eE_0}{2}$, $\rho=\frac{I_\infty}{I_d}$, $\delta=\frac{k_0x_0r_en_e^4k_BT}{2e}$, $A=\sqrt{\frac{n_e}{2\eta_0I_d}}\phi$ and $\eta_0=\sqrt{\frac{\mu_0}{\varepsilon_0}}$, x_0 is the width of the soliton beam, x is the position and ξ is the propagation distance. Eq. (32) is the modified nonlinear Schrödinger equation (MNLSE)[16]. Solitons can bend depending on the parameter δ.

2.4 Linear stability Analysis

The parameters α, β are used to control generation of solitons and the parameter δ is associated to the diffusion term. By neglecting β and δ, Eq. (32) becomes[12]

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Volume 3, Issue 5 May 2021, pp: 559-564 www.ijaem.net ISSN: 2395-5252

$$i\frac{\partial}{\partial\xi}A=-\frac{1}{2}\frac{\partial^2}{\partial s^2}A+\alpha\frac{(\rho-|A|^2)}{1+|A|^2}A. \quad (33)$$

For dark screening solitons, I_∞ must be finite hence ρ also must be finite. Eq. (33) will guide us to the soliton formation. If we use the ansatz as perturbation solution:

$$A=(A_0+\psi)e^{i(ks-\omega\xi)}, \quad (34)$$

where $\psi=\phi(\xi,s)$, A_0 is the real constant amplitude of waves, ψ is the perturbation and ω is the phase of the perturbed solution. Replacing Eq. (34) in Eq. (33) and neglecting the terms containing higher order than one in ψ and ψ^* of the new equation obtained after replacement, it leads us on equation describing the dynamics of the perturbation,

$$i\frac{\partial\varphi}{\partial\xi}+ik\frac{\partial\varphi}{\partial s}=-\frac{1}{2}\frac{\partial^2\psi}{\partial s^2}-\alpha\left[\frac{\rho A_0\psi}{(1+A_0^2)^2}+\rho\frac{A_0^2\psi*}{(1+A_0^2)^2}+\frac{A_0^2(\psi+\psi^*)}{(1+A_0^2)^2}-\frac{A_0^4(\psi+\psi^*)}{(1+A_0^2)^2}\right]=0. \quad (35)$$

We can simplify this notation in order to facilitate the process of perturbation.

$$i\frac{\partial\psi}{\partial\xi}+ik\frac{\partial\psi}{\partial s}=-\frac{1}{2}\frac{\partial^2\psi}{\partial s^2}-\Delta(\psi+\psi*), \quad (36)$$

where

$$\Delta\equiv\alpha\left[\frac{A_0^2}{(1+A_0^2)^2}+\rho\frac{A_0^2}{(1+A_0^2)^2}+-\frac{A_0^4}{(1+A_0^2)^2}\right]=\alpha(1+\rho)\frac{A_0^2}{(1+A_0^2)^2}. \quad (37)$$

Let us take perturbation amplitude as

$$\psi=\psi_1+i\psi_2, \quad (38)$$

where ψ_1, ψ_2 ε *, and substitute Eq. (38) in Eq. (36) leads us to

$$i\frac{\partial}{\partial\xi}(\psi_1+i\psi_2)+ik\frac{\partial}{\partial s}(\psi_1+i\psi_2)=-\frac{1}{2}\frac{\partial^2}{\partial s^2}(\psi_1+i\psi_2)-\Delta(\psi_1+i\psi_2+(\psi_1-i\psi_2). \quad (39)$$

If we separate realand imaginary parts from Eq. (39) gives [12]

$$\frac{\partial}{\partial\xi}\psi_1+k\frac{\partial}{\partial s}\psi_1=-\frac{1}{2}\frac{\partial^2\psi_2}{\partial s^2}, \quad (40)$$

and $$\frac{\partial}{\partial\xi}\psi_2+k\frac{\partial}{\partial s}\psi_2=-\frac{1}{2}\frac{\partial^2\psi_1}{\partial s^2}+2\Delta\psi_1. \quad (41)$$

The following expressions can facilitate to know the variation of perturbation.

$$\psi_1=R_e\left(U_1e^{i(qs-\Omega\xi)}\right)=U_1\cos(qs-\Omega\xi) \quad (42)$$

$$\psi_2=\mathrm{Im}\left(U_2e^{i(qs-\Omega\xi)}\right)=U_2\sin(qs-\Omega\xi) \quad (43)$$

where U_1 and U_2 are possible amplitude of perturbation, U_1, U_2 ε, $(qs-\Omega\xi)$ is the modulation phase, q is the phase number and Ω is the frequency modulation. Replacing Eq. (42) and Eq. (40) and then after we simplify, it gives,

$$(\Omega-kq)U_1=\frac{1}{2}q^2U_2. \quad (44)$$

Let us substitute Eq. (42) and Eq. (43) in Eq. (41) and simplify, we obtain

$$(\Omega-kq)U_2=\left(\frac{1}{2}q^2-2\Delta\right)U_1. \quad (45)$$

Eq. (44) and Eq. (45) are forming the system of equations which are leading to determinant equal to zero, for getting more solutions

$$(\Omega-kq)^2=\frac{1}{4}q^2(q^2-2\Delta). \quad (46)$$

Eq. (45) is known as dispersion relation.

2.5 Modulation instability

From Eq. (46), it is possible to obtain the parameter domain where occurs modulation instability. Modulation instability is the process in which weak perturbation can grow exponentially when it propagates in media which nonlinear. When a continuous wave propagate in nonlinear media, at some distance, it starts to be unstable, which causes the disintegration of plane waves into a larger number and leads to the breaking up into many filaments of optical pulses. Modulation instability is the result of reciprocity between nonlinearity of media and the dispersion of waves [12. During the modulation instability process the waves disintegrate into a large number of waves which results in the creation of soliton waves[1]. Modulation instability is used to study some physical behaviors of systems such as fluids, plasma, optical fibers, etc .If

$$q^2-2\Delta>0, \quad (47)$$

from dispersion relation in Eq. (46), the frequency of modulation is a real number. It is leading to non-instability which is not concerning our work. If

$$q^2-2\Delta<0, \quad (48)$$

The frequency of modulation has real and imaginary parts, when imaginary part of the non-modulation frequency in non-nil, conducts to the exponential increase of the amplitude of the perturbation, such case, we have

$$\Omega=\pm i\frac{1}{2}[-q^2(q^2-4\Delta)]^{\frac{1}{2}}+kq. \quad (49)$$

The imaginary part of the modulation frequency Ω causes an exponential growth of amplitude. During this process of instability of modulation, solitons are formed. The modulation instability gain g has dependence on q, α and ρ,

since it is known from that $\Delta = \Delta(\alpha, \rho)$. The modulation instability gain can be defined from the imaginary part of the modulation frequency as

$$G = |\operatorname{Im}\Omega| = \frac{1}{2}\sqrt{-q^2(q^2 - 4\Delta)} \quad \textbf{(50)}$$

III. CONCLUSION AND RECOMMENDATION

The full model includes four Parameters that contribute to the nonlinearity of the system, and therefore may all impact the soliton formation. However, to simplify the calculations, we considered only two parameters, α and ρ. It will be important to consider the other parameters, β and δ in further research work and use simulation in order to show how these parameters have effect on soliton formation.. This may allow to check how solitons bend when they are propagating in nonlinear media. Wavenumbers satisfying $q^2 < 4\Delta$ are unstable, and can cause the formation of solitons. We have also determined the modulation instability gain which appears to depend on the parameters α. For a given Δ, the maximum gain is obtained for the wavenumber $q_m = \pm\sqrt{2\Delta}$.

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